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THE MATHEMATICS TEACHER

Volume XLIII OCTOBER 1950 Number 6

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OFFICIAL JOURNAL OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
1201 [illegible], N.W., Washington 6, D.C.

Entered as second-class matter at the [illegible] Act of [illegible], authorized [illegible] special rate of postage provided [illegible] Act of February [illegible], authorized [illegible] 412 P. L. & R., authorized March [illegible]

THE MATHEMATICS TEACHER

Official Journal of the National Council
of Teachers of Mathematics

Devoted to the interests of mathematics teachers in Elementary and Secondary Schools, Junior Colleges and Teacher Education

Editor and Chairman of the Committee on Official Journal—E. H. C. HILDEBRANDT, 212 Lunt Building, Northwestern University, Evanston, Illinois.

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All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, membership dues in The National Council of Teachers of Mathematics, subscriptions to THE MATHEMATICS TEACHER, and notice of change of address should be sent to:

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1201 Sixteenth Street, N.W., Washington 6, D.C.

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1950–

THE MATHEMATICS TEACHER

Volume XLIII Number 6

The President's Page

First, I wish to pay tribute to our highly esteemed Past President, Dr. E. H. C. Hildebrandt. For many years he has given unstintingly of his superior abilities and apparent inexhaustible energy to the work of the National Council. As president during the past two years, he has brought into focus many important services of the Council. In getting things done, he has been able to enlist the cooperation of a vast number of members and has made valuable contacts and relationships with other national organizations. With his efficient leadership, the National Council has shown itself to be what its name implies, a national organization of great influence in mathematics education. Dr. Hildebrandt has made for himself a very enviable record as a conscientious and unselfish leader in the educational world. We admire him for his honesty, sincerity, and integrity. We have been proud to claim him as our leader and we are fortunate and happy to retain his services as Editor-in-Chief of our official journal, The Mathematics Teacher.

While I fully appreciate the honor that comes to me by being chosen President of your National Council, I am much more impressed and concerned about the work that is to be done. Many fine things have been initiated during the past few years. These must be continued. Our National Council is growing very rapidly and all of us must help our thriving organization to keep pace with the opportunities of service that lie ahead.

I am a classroom teacher and proud of it. I think I know the kind of help the mathematics teacher needs. Although a teacher of high school mathematics, I have worked with elementary teachers and am aware of their problems. My summer college teaching keeps me alert to the problems there. Supervising student teachers makes me conscious of the needs of the beginning teacher. Serving on various committees, both local and national, affords me a good perspective of the situation as a whole. My work over the past twenty years with the National Council has given me a greater appreciation of the help such a national organization can be to the teacher. As your president, I intend to give my very best to the work of the National Council. I want to see it become even more important as a service organization for the improvement of mathematics teaching.

Since the President is privileged to have this page each month, it will be my plan to bring to your attention certain endeavors of the Council and to enlist your help and cooperation. You should know about the things that the Council is trying to do, the progress being made, and you should be given the opportunity to help. As the Council extends its services, memberships will increase. As memberships increase, services will grow. Every member should feel that he has a real part in the work and activities of the Council. If there is something you want to do to help, please do not hesitate to offer your services. If you have suggestions, please send them to us. This is your Council and we want you to have a real part in it. Feel free to write to me any time.

H. W. Charlesworth, *President*

An Engineer Looks at Mathematics Teaching*

By EVERETT S. LEE

Executive Engineer, General Engineering & Consulting Laboratory, General Electric Company, Schenectady, N. Y.

YOUR good president has asked me to speak to you on the subject "An Engineer Looks at Mathematics Teaching." I was given this honor as I had been privileged to be with Professor Rankin at his Institute at Duke University for Teachers of Mathematics, where we have had such good times together. Through the contributions of Professor Rankin and his associates and through the contributions of your own National Council the work being done in mathematics throughout our land is enriched and a true concept of the proper place of mathematics in our educational program is obtained by all who have a responsibility in this field—teacher, principal, professor, and dean, together with all those who need and use mathematics in their daily living and work. This is everybody.

Throughout my engineering life I have been privileged to have been associated with one of our large laboratories in the General Electric Company, our General Engineering and Consulting Laboratory. This is a measurements and engineering development laboratory, serving the entire engineering staff of the Company, together with associated manufacturing. It takes the new knowledge from the scientists of the Research Laboratory and fashions it into metal and material, and serves today with a heritage from the past of those master minds, Dr. Steinmetz, Dr. Alexanderson, and Dr. Lewis Robinson. Because of our associations with the other laboratories of the great industries and schools and government of our land, and with their engineers and scientists, I trust that in speaking of our experiences I can also be speaking for them, although this is by association only and not as their direct representative.

To many people the inhabitants of a laboratory are "nuts," different from everybody else and limited in their concepts of life. This is not so. Having established this fact, I want to add quickly that the techniques in the engineering laboratory as regards mathematics extend out into the engineering field and into the life of engineers broadly, both in the technical and general aspects of their living. So it also is with scientists. Within and beyond the laboratory there is little fundamental difference as regards that product in which you are making your day-by-day contributions—mathematics.

We all need mathematics. Everybody has to know how to add—though few can add a column of figures accurately and quickly. Your teaching should be undergirded in this regard; addition should be made a continuing part of your curriculum. So should subtraction, multiplication and division.

For the scientists and the engineers algebra and geometry are also essential, principally for two of the mathematical concepts—the equation, and the plotting upon squared paper of the equation. Here are fundamental concepts of the greatest import. They coordinate immediately with what I feel is the greatest attribute of mathematics, that it is—the means whereby phenomena are visualized for practical use.

If there is any thought in the mind of the student who does not understand mathematics and its place in his life, and the reason for him having to master it, it is that, first, he doesn't understand where he will use it and, second, it is too hard. I am confident that, if he can see where it will be useful to him, he will give of his abilities to its mastery.

* Presented at the Banquet at the 28th Annual Meeting of the National Council of Teachers of Mathematics, Chicago, Illinois, April 14, 1950.

You may put mathematics first—but I put it second. To me the phenomenon is first and mathematics second, but we need them both. Both are essential, so whether first or second is important only as a factor in establishing the proper concept.

Ma does not go to the grocery store to add up a column of figures—she goes because Pa and the kids and she must eat. Hunger is the phenomenon—the mathematics in the grocery store comes as a direct consequence of that phenomenon.

And so it is in our engineering and scientific lives. The phenomena which your associates in the school of science teach in the natural sciences are fundamental. When the phenomena are understood, completely or even partially, there is the urge to apply them. Here the mathematical equation comes into play, for the expression of the scientific phenomenon in one line is the equation of mathematics. This requires both scientific and mathematical knowledge—the scientific to visualize the phenomena and the mathematical to put the visualization into equation form.

Then follows more mathematical knowledge in the solution of the equation to result in an expression of variables. Substituting real values for the variables allows the phenomena to be resolved into practical use.

This is the picture which the science teacher, together with the mathematics teacher, can bring to the young student better than anyone else in any other way. In high school the presentation will have to be confined to arithmetic and algebra and geometry, for these the student has had. The application here will please him since he is using mathematics he has learned previously and since he is finding in science a use which he could not advance in any other way. Thus his scientific knowledge and his mathematics knowledge will go forward together to bring him ability in visualizing phenomena and in using his mathematics to make those phenomena useful.

You who hold the opportunity for bringing this concept to the young scientist and to the young engineer, and who will help him to nurture it, will be putting into his scientific and engineering life one of his most valuable acquisitions. It is what made a Steinmetz—it is what has made many a famous engineer and scientist. It is what you can provide for a life of greater usefulness for your students as you live with them today.

Now I visualize that, as you bring the inspiring opportunities of mathematics to your students, you find some of them keenly interested to go on and on to find more applications of what you are unfolding to them. Among these are the engineers of tomorrow. We are especially interested in them as they continue to learn and to show an ever-increasing interest in the new and a growing desire to use it. These are the boys—and the girls, too—who will be entering the engineering schools of our land to learn more that is new, and to build on that of which they have knowledge. In the days to come they will be entering their engineering careers to continue to bring to mankind even greater services than he has yet had provided for him.

Some of these youngsters with whom you will be privileged to spend a few years, and who will be privileged to spend a few years with you, will be especially keen in scientific perception and observation. They will move ahead into unknown fields faster than their fellows. From these will come the scientists of tomorrow, who will continue throughout their lives to bring new knowledge into being—new knowledge of our universe and of all that is therein—that this new knowledge may be made available for the engineer to fashion into material for the use of mankind.

You have in their formative years the boys and girls from whom will come the scientists and the engineers of tomorrow. What greater wealth could there be! You shape and guide them in their beginning years—yours is a responsibility of the

greatest of service—yours is a responsibility to all mankind.

Every material thing we have comes from the scientists and the engineer. This paper from which I am reading, the press on which it was printed, the train which brought it, all came from the scientist and the engineer. The men of science and engineering gave us the house in which we are, the furnishings, the lights, the street outside, the buildings of the city, the telephones, the telegraph, the movies, radio, television. The farm equipment on the farms, the roads, the automobile, the airplane, the electric power for home and industry were created by the scientist and the engineer. The tanks, guns, planes, ships, and ammunition of our war production all came from the scientist and the engineer. The opportunities for all of the people to work in peace have come from the scientist and the engineer. The opportunities for all the peoples to work in peace in the future will come from the scientist and the engineer. You have these scientists and engineers in their youth; yours is the greatest of responsibilities.

Now the line from the research laboratory and the model shop of the engineer to the fact of the product in the hands of the people is a long one. In it stands the great industrial leaders of our land who have led us to bring forth large companies, and the medium-sized companies, and many, many small companies which are our great industrial strength in this country to progress the products of the engineer. In it stand the workers in the shops who with the proficiency of their hand and eye operate the machines to turn out the products of the engineers for the people to have and to use. In it stand the salesmen who separate the dollars from the people for the products, and the products thus finally get to the people for their use and enjoyment. This is a long and noble procession.

Now there is another group of people entwined in this procession. There are the doctors who keep us in good health. There are the lawyers who keep us in line and keep us out of jail. There are bankers who provide the wherewithall to give us a start and to help in keeping us going. There are the ministers who guide us in paths of righteousness. There are the boys in the Service who are keeping us at peace. And last but by no means least, are you, the school teachers, who impart knowledge and wisdom and understanding all along the line to all those entering the procession that it may be continuous and of ever increasing ability. You undergird the entire procession. You maintain it. You are responsible for its advance. You have a responsibility which is as great as you can fulfill. You leaven the process. Without you the process stops.

Mathematics has a vital place in this picture. Without it everything would stop. There would be no trade. There would be no production. There would be no measurement. There would be no statement of result. There would be no communication of ideas. There would be no evaluation of project. There would be no coordination. There would be no fact.

I do not presume to tell you how to teach mathematics, but I do want to emphasize that when you can get your young students to see and visualize phenomena in nature and can bring to your students the importance of mathematics in this picture, then you are laying the foundation for scientific and engineering advance. Lead them to try to understand these phenomena—to cogitate upon them—to build with them upon that which they already know. Let their imagination run wild with the opportunities for the use of the phenomena. This is what has brought us to where we are today, and will carry us to better things to come.

In saying this I recognize that most of your young people in mathematics will not become either scientists or engineers—but they will become doctors or bankers or lawyers or politicians or merchants or trades people or artisans or farmers or clerks or housewives to take their place

in the great procession of what we are all doing together. All of these need and can use the same training; it is fundamental to the existence of our people. In fact, it is even more important that those who use the products of the scientists and the engineers use them for good and not for evil. This is the desire of the scientist and of the engineer; this should be the motive of those who use their products.

I know of no surer way to bring the teachings of right and wrong to young people than by disclosing the laws of natural science and of mathematics, their exactness and their constance of behavior, as contrasted with the behavior of man. And I know of no group better equipped to do this than those who teach science and you who teach mathematics to the youth of the land. I speak from personal experience in my own case, and I have been privileged to have been associated with many engineers and scientists who, I find, understand these fundamentals. For this I have naturally given credit to you who play the beginning role in their scientific lives. If what I have said is not new, I say again for emphasis that it is still the truth. If it is new to some of you, I say it with a conviction that it is essential to our continued advance, that it be for great good.

And together with all these, I trust you may, as there is opportunity for you to do so, bring to your boys and girls the knowledge and the conviction that there must be a power behind all nature greater than the power of man, a mind greater than the mind of man, and a life greater than the life of man; for unless it were so, these things could not be. As you bring to the boys and girls the knowledge that the laws of nature and of mathematics cannot be broken, so there should be the knowledge that the laws of human conduct cannot be broken without redress.

Well, I left you with algebra and from it I have brought in all of the fundamentals I know as they exist in this picture.

Trigonometry is next, and then comes college with analytical geometry, descriptive geometry, the calculus, integral and differential and differential equations. Most stop here, but some as they have ability continue on. It is good that those who can, do travel far along the mathematical boulevard, always keeping in mind the scientific phenomena which they can the better clarify and unlock for the use of man as they progress.

In this regard I asked one of our Advanced Engineering Program directors for a few applications of mathematics beyond the usual, and note the practicability of the problems to which they are applied. Here are some he offered.

Bessel's Functions are applied to the problem of the cooling of a gas-turbine wheel. This new power development of the past few years has been made possible by the availability of higher temperature materials to give efficiencies in prime movers beyond any previously attained.

Bessel's Functions also arise in expressing a frequency modulated signal as an infinite series to analyze the fidelity of sound transmission of this communication method. Bessel's Functions occur also in the study of cylindrical wave guides, and in calculations of drag on finite supersonic air foils.

Tensor analysis is another new field beyond the usual. Mr. Gabriel Kron has done a great deal of work on the equivalent circuits of electrical machines, that their design may be more efficient, their performance better, their cost less. Tensor analysis has also been applied to the solution of steel mill control problems. One unusual problem for solution by tensor analysis was that of the mechanical linkage in a gun-fire computer subjected to a sudden acceleration caused by gun recoil.

Conformal transformations have their place also in the most practical of problems. In the problems of the fringing flux in a direct-current machine, using a Schwartz-Christoffel transformation, it was possible to calculate the amount of

flux between the main pole and the interpole.

The problem of the amount of heat conducted through a corner of a thick rectangular duct may also be solved by using a Schwartz-Christoffel transformation. And the problem of lift on air foils may often be solved by using a transformation to transform the air foil to a simpler shape.

A simple problem in solving Legendre Polynomials is to determine the field pattern of an antenna made of two hemispherical shells closely spaced.

One application of elliptic integrals is in the calculation of a high-frequency inductance and radiation resistance of a coil; another is the calculation of viscous flow in liquids for non-turbulent motion.

The theory of characteristics is used in the solution of problems in gas dynamics involving two-dimensional and axially symmetric supersonic flow.

Considerable application is made of the methods of operational calculus using both the LaPlace transformation and Heaviside's Operators. They are used in problems involving servo-mechanisms, transmission line systems, transient and steady state performance of electrical machinery, and electrical and mechanical networks. To understand these operational methods, the engineers using them have become familiar with complex variable theory.

These concepts, among the advanced of the mathematics, are applicable to many problems—problems which need the best of scientific and engineering knowledge and ability for solution. Most people little realize how much knowledge is put into engineering designs. You hold the key to the mathematics which produces results that can be achieved in no other way except occasionally by hunch or by unusual perception. These are the exception, however, not the rule. And, of course, there are some who will advance mathematics just for mathematics. These we need also for their great power.

In between the early fundamental concepts of the student in the early school and the applications of the most advanced mathematics reserved for the relatively few, an enormous amount of mathematics is used by the thousands of engineers and scientists in their daily engineering lives. This also you supply, and the greater its understanding and its application, the greater is your contribution and the more effective are you progressing your responsibility. So effective has it been that I hesitate to advance anything new or even to try to suggest any detailed opportunities for improvement. Of course, there are many opportunities—for there is always a better way—and I trust you are progressing these in the excellent study groups which are so potent a part of your institutes and of your Council meetings.

I do want to emphasize one phase of fundamental importance—that when appreciation and understanding of the phenomena involved are brought together with the mathematical knowledge, then the two are progressed together to a result attainable in no other way with comparable exactness and certainty of attainment.

I was privileged to have as one of my teachers that truly great teacher, Dr. E. J. Berg. He was a great teacher. He loved his boys and worked hard with them to bring to them a realization of the wonders of nature and of the opportunity that lay ahead for those who could visualize phenomena and apply mathematics thereto, and thus achieve a final result having practical application. If you are not familiar with his method I refer you to his text, *Electrical Engineering*, by Berg and Upson. This book is full of what I have been saying, and I know its great power from practical experience.

One of the most fascinating problems was that of the short circuiting of an alternating-current generator. I can see Dr. Berg to this day, as he stood at the blackboard, describing to us the phenomena of short-circuit, what happened during the transient period from the steady state of

normal power operation to the steady state of short-circuit—how the field current died down, how the flux decreased, what factors were affecting the induced electromotive force, and finally arriving at a differential equation which could be solved to obtain an equation to give the instantaneous value of the armature current. In this came a recognition of the phenomena to give data for obtaining the values of the integration constants which are so necessary. And then, after obtaining the final equation, work, work, work, substituting in the equation to find the values of the current under many different circumstances. And plotting the results on squared paper to see the phenomena in all of its beauty as the curves unfolded on the squared paper—hard work that required knowledge of the phenomena, knowledge of mathematics to write the equations, knowledge of mathematics to solve the equations, and more hard work to evaluate the phenomena as it appeared on the squared paper.

Vast mathematical machines do much of this work for us today—and do more that ever could be done by human calculation. Nevertheless, always remember that without the knowledge in the brain of the engineer and of the scientist of the phenomena and of the applicable mathematics—without the engineer behind and within and surrounding the machine—the machine cannot function. The output is multiplied only because the engineer-scientist-mathematician is there to prepare it for the machine originally and to use it after it comes from the machine. Without the engineer—the machine is just a machine.

There is no decrease in these fundamental needs—but rather there is an augmentation necessary. In every field there are still unsolved problems. The pressure for greater output, for more versatile and simpler performance, for greater efficiency, for lower cost, demands a better and keener understanding of the phenomena involved and a better and keener understanding of the mathematics which can be applied to the phenomena to produce the desired result. You who hold the key to the unfolding of these truths to the youth of our land may glory in your past achievements and may gird yourselves ever more faithfully to live to inspire in your students a love for these attainments which will bring to mankind greater productivity and greater happiness.

I firmly believe a good dose of this doctrine is what the world needs from our good friends in other than science and mathematics and engineering of our schools. Then there will be brought into the lives of men that appreciation of the laws of nature which the scientist and engineer and mathematician have—that you pay for what you get and that you do not get something for nothing.

Mathematics Kits

The mathematics kits prepared jointly by the National Council of Teachers of Mathematics and Science Service continue to be in demand by mathematics teachers. The second kit in this series was issued in April. It is known as Straight Line Unit, No. 113. This kit deals with the interesting and fundamental problem of drawing an original straight line. It shows how the problem may be solved by means of linkages. The kit contains materials for making four workable models and includes an explanatory leaflet.

Also there are still a few kits left of Geometric Models Unit, No. 102. The kits sell for 50¢ each or 3 for $1.00, and may be assorted. For the present the distribution of the kits is being handled by the chairman of the kit committee, Professor M. H. Ahrendt, Anderson College, Anderson, Indiana, and orders should be sent to him. Please send payment with each order.

Adapting the Curriculum in Primary Arithmetic to the Abilities of Children

By HERSCHEL E. GRIME

Directing Supervisor of Mathematics, Board of Education, Cleveland, Ohio

THAT the abilities of children vary is a generally accepted fact. It follows then, that children will not all learn the same things at the same rate, neither will the quality of the work done be the same for all of them. In planning the curriculum in any subject, provision should be made not only for differences in the learning rates of children, but also for differences in the standards of achievement expected of them.

To provide adequately for these differences in children requires that the curriculum be organized and the classroom procedure planned so that each child may be taught either as an individual or as one of a small group having relatively similar abilities. At the same time, the requirements for a given period of time should be in keeping with the child's learning rate. In Cleveland this has been accomplished by dividing the arithmetic of the entire primary division into fourteen units of work or levels of achievement. These levels are not all of equal subject content. They require different amounts of work and time for mastering. Each is, however, a definite step or group of related and sequential steps in the development of arithmetical ability.

The entire course is organized in such a way that an individual pupil, or a small group of pupils with relatively similar abilities, may progress at his own rate from one level to the next without failing for a semester or a year. As soon as the pupil, or a group of pupils, has mastered one level, he begins the next, regardless of the time of year. When school opens in the fall, each child begins just where he stopped when school closed in the spring. Thus the progress from level to level is natural, orderly and systematic.

Since success in arithmetic depends upon the mastery of a cumulative series of skills, the specific objectives for children of all abilities are the same for each particular level. Provision for individual differences is made in several ways. The choice of activities and problems, the method of presentation, the type and amount of practice, the degree of mastery required, and the length of time spent on a level may be varied by the teacher to suit the abilities and the needs of her pupils.

Provision for enriching the experiences of the primary child has been made in various ways. In the first place, no child is held back if his natural ability and interest lead him to progress at a rate exceeding that of the other children in his age group. The bright child is given a more thorough understanding of numbers and number relations as well as a wider application of the use of numbers in solving problems than is given to the slower-learning children. Furthermore, group activities and planned units of work provide many opportunities for participation in quantitative experiences that serve as a challenge to even the brightest child.

Children who make normal progress in arithmetic complete the work of two levels in the kindergarten and four levels in each of the three years of the primary division. Children who do not learn arithmetic readily require a longer time for completing a level. At the same time, the quality of their work may be considerably lower than that of children who progress at a normal rate. Some children may require seven or even eight semesters beyond the kindergarten before they are ready for the arithmetic of the upper elementary division.

In arithmetic just as in reading, over

far too long a period, teachers have plunged beginning pupils into the mechanics of the subject before providing a meaningful background of quantitative thinking and judging. As a result of recent studies, evidence is available which seems to prove what we believed—that if more time and effort is spent in the early levels in developing a rich background of quantitative experiences, the child will make more rapid progress in the later levels than he would have if he had followed the more formal program. Furthermore, these early experiences will develop the ability to use numbers efficiently in meeting situations that arise in everyday activities.

In accordance with this belief, a reduction has been made in the amount of formal work in the early primary levels. Changes have been made both in the type of arithmetic being studied and the level in which the different topics are introduced. The work of the first six levels is very informal and often incidental on the part of the child. The teacher, however, has definite objectives in mind. The early levels deal chiefly with the building of concepts, the understanding of number relations, and the quantitative vocabulary needed by the children in solving the problems that arise in their everyday experiences.

It is not until Level VII, about halfway through the primary division, that the children are required to master any number facts. In this level they are taught the easier addition facts if they have not already actually learned them through use in many informal situations in the preceding levels. From Level VII on, the work becomes more formal in character. New facts and new steps in the three fundamental processes, addition, subtraction, and multiplication appear gradually in the later primary levels. Meaning before practice is stressed throughout.

So that every teacher in the primary division may know just what to teach in a particular level, a Course of Study in Arithmetic for the Primary Division has been published. The materials presented in this course of study are the result of careful study and experimentation over a period of many years. Practically every primary teacher and principal in the elementary schools of Cleveland contributed suggestions or participated in some phase of the study, planning and experimentation that was involved in its development.

This course of study states, in simple language, the philosophy underlying the teaching of arithmetic in the primary division. It is intended as a guide and a source of help in making arithmetic vital and meaningful to the primary child. The objectives for each of the fourteen levels are listed with many suggested procedures for teaching each of them. Liberal use has been made of illustrations to clarify procedures where necessary.

One section is devoted to units of work or projects that can be used to make arithmetic meaningful and to show how it can be used in everyday classroom activities. Another section is devoted to games and devices through which the child is led to discover the meaning of number facts and relations, or which may be used for practice.

The evaluation of what the child has learned in any particular level must begin with a consideration of what we have tried to teach him. In the primary levels this includes much more than skill in computation and in problem solving. It includes an understanding of arithmetical concepts and processes, a vocabulary of useful quantitative terms, and an understanding of units of measure and the process of measurement. Above all, it includes the disposition and the ability of the child to use numbers and number processes as needed in his everyday experiences.

While many of the outcomes of the teaching of arithmetic can be measured reliably and accurately by pencil-and-paper tests, some of them cannot be measured in this way. Recognizing this fact, the teacher in the primary levels

uses such tests but supplements them with other procedures such as the observation of the pupil at work and the examination of the product of his work.

The ability to use arithmetical ideas and skills in the ordinary activities of the classroom and in planned units of work is the best kind of evidence that the pupil understands what has been taught. This evidence is noted by the teacher as she works with her pupils from day to day. By means of progress charts of various kinds she keeps a record of the growth in arithmetical ability of each child.

To aid the teacher in determining whether the pupils in a particular level have a thorough grasp of the concepts, skills and understandings that have been taught, a series of tests, the Cleveland Achievement Tests, have been developed. As these tests are intended to measure all the objectives of the level, the content of each follows closely the Cleveland Course of Study in Arithmetic. The beginning primary tests deal mainly with the child's number background and vocabulary. In these early levels the test items consist of pictures and illustrations, with the directions for each item read by the teacher. Since the purpose of teaching arithmetic is to help the child solve his everyday problems, simple one-step problems are introduced early in the series. Abstract computations appear first in the test for the tenth level.

Under this plan of organization, which has been in effect in the Cleveland Schools for fifteen years, the pupil who learns arithmetic readily is not held back and compelled to progress at the same rate as the slower learning members of the group but is allowed and encouraged to proceed at his own rate. The child who has difficulty in learning arithmetic is not discouraged and defeated by the imposition of standards of time or achievement which he is unable to attain. The slow-learner, as well as the more rapid, is allowed to progress at the rate best fitted to his ability. The child's achievement in arithmetic is considered satisfactory if the progress made is in keeping with his ability.

Nominations for Members of the Board of Directors

According to Article III, Section 7 of the By-Laws of the National Council of Teachers of Mathematics, "at least two months before the date of the annual meeting, all members shall be given the opportunity through announcement in the official journal to suggest by mail for the guidance of the Directors a candidate for each elective office for the ensuing year."

The 1951 ballot will present candidates for the following offices:

A Vice-President representing the field of college mathematics;
Three members of the Board of Directors.

The candidates for Directors may be teachers in schools at any level. The present geographic plan requires that only one of the nine Directors may come from one state. This limitation does not apply to the officers. Hence, the following states will not be represented on the Ballot for the three Directors: Louisiana, Maryland, Massachusetts, Minnesota, New Jersey, or Wisconsin.

The Nominating Committee will accept suggestions for candidates until November first. A brief statement of the candidate's qualifications and information regarding his participation in local or national mathematical affairs should be included.

Suggestions may be sent to Miss Ona Kraft, Vice-Chairman on the Nominating Committee, Collinwood High School, Cleveland, Ohio or to any of the following members of the Nominating Committee:

George E. Hawkins, Lyons Township High School, LaGrange, Illinois; E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois; Elenore M. Lazansky, Claremont Junior High School, Oakland, California; Edwin W. Schreiber, Western Illinois State College, Macomb, Illinois; Carl N. Shuster, New Jersey State Teachers College, Trenton, New Jersey.

Teaching for Generalization in Geometry*

By Frank B. Allen
Lyons Township High School, LaGrange, Illinois

Teaching for generalization seeks to chart the forest as well as to scrutinize the individual trees. Such instruction should be guided by a panoramic view of those ancient verities which compose the subject matter of classical geometry. The type of view obtainable, for example, is like that from an aerial photograph. If you have examined such photographs, you have, no doubt, observed that they not only show a surprising wealth of detail within an area but they also show the relation of that area to surrounding territory. In such a picture, each detail of the landscape is rendered more significant by the fact that it is seen in its proper position with respect to the entire area.

The analogy may be noted when we attempt to transform this idea into a more intelligible description of the kind of instruction I have in mind. For the purposes of this discussion such teaching can best be characterized in terms of three major objectives.

1. Teaching for generalization attempts to stress the relations between geometric facts which serve to bind them together into logically coherent groups, thereby rendering the individual facts more significant.
2. It seeks to develop an appreciation of the power and scope of a general statement by presenting it as a unifying principle linking together special cases which at first seemed totally unrelated.
3. It strives to find principles which may be effectively applied for the purpose of deriving valid generalizations.

These three objectives, lofty and ambitious as they are, are not meant to be all inclusive. They are perhaps sufficient to indicate that teaching for generalization is one aspect of teaching for transfer. There are some who believe that the very survival of geometry in our secondary schools depends on the extent to which it can be demonstrated that such teaching is possible. Of course, it has long been our conviction that demonstrative geometry provides a most appropriate setting for the study of the general principles which govern logical thinking. The time has come for us to be explicitly outspoken about this idea. We must advance our arguments vigorously and with the clarity and force necessary to make them compellingly convincing.

It is in this connection that I venture to suggest that it is only when we attempt to generalize that we fully realize the potentialities of our subject. As Christofferson says, "Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can develop the power to think clearly in geometric situations and to use the same discrimination in non-geometric situations; if it can develop the power to generalize with caution from specific cases and to realize the force and all inclusiveness of deductive statements."[1]

Now let me ask a very difficult question. How can we teach for generalization? Granting that it is desirable or even essential, how can we do it? Any worth-while answer to this question must be specific. It must utilize *specific cases* to illustrate *general techniques*. When one attempts to become specific—when he ventures to describe actual class room procedures—he must realize that he has entered an intensely controversial area. I am reluctant to enter this area but I see no other way to specifically describe the procedures I have in mind.

In order to answer the above question as best I may, I shall break it down into

* Read at the Ninth Christmas Meeting of the National Council of Teachers of Mathematics, Columbus, Ohio, December 29, 1948.

[1] H. C. Christofferson "Geometry Professionalized for Teachers" (Oxford, Ohio: published by author, 1933), p. 28.

three questions corresponding to the three previously stated objectives of generalized instruction. These three questions will serve as an outline for the balance of this discussion.

1. How can we stress the relations between geometric facts—and how does such emphasis render the individual facts more significant?
2. How can we develop an appreciation of the power and scope of a general statement?
3. What principle may be utilized for the purpose of deriving valid generalizations?

Perhaps a partial answer to the first question may be found in the use of *the multi-converse concept*. The definition of converse which is most useful for this purpose is that stated by Lazar as recently as 1938 in a thesis bearing the title "The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry."[2] A converse of a theorem, says Lazar, may be obtained by interchanging any number of conclusions with an equal number of hypotheses. This definition requires a careful listing of the conditions in each part of the theorem. We might find on analysis that there are three conditions in the hypothesis and two conditions in the conclusion. This can be be shown schematically as follows:

$$\left.\begin{matrix} A \\ B \\ C \end{matrix}\right\} \longrightarrow \left\{\begin{matrix} D \\ E \end{matrix}\right.$$

Once we have our theorem in this form, together with a corresponding diagram, we no longer rely upon the verbal statement as a basis for formulating converses. Instead we apply Lazar's definition to the arrangement represented here. Students learn to do this very quickly and they are invariably astounded at the number of converses obtainable from one key theorem. For example, a theorem of the type shown here may have 9 distinct converses. But, as Lazar says, the mere numerical increase in the number of converses is by itself of little importance. The important consideration from our viewpoint is the fact that the multi-converse concept enables *the student* to trace a *strong logical connection between theorems*. In this way an entire unit may be organized and headed up by one key theorem from which all the other theorems in the unit may be obtained by the use of Lazar's definition. Of course, any theorem in the set may be selected as the key theorem.

For example, consider the key theorem: If a quadrilateral has both pairs of opposite sides parallel, then the opposite angles are equal and the opposite sides are equal.

On analysis we have:

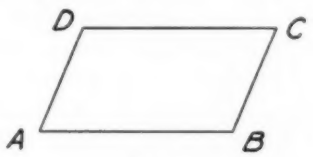

FIG. 1

$ABCD$ is a quadrilateral

$$\left.\begin{matrix} \text{H1. } AB \parallel CD \\ \text{H2. } AD \parallel BC \end{matrix}\right\} \longrightarrow \left\{\begin{matrix} \text{C1. } AB = DC \\ \text{C2. } AD = BC \\ \text{C3. } \angle A = \angle C \\ \text{C4. } \angle B = \angle D \end{matrix}\right.$$

If we interchange the first hypothesis with the third conclusion we obtain a theorem which can be stated thus: "If a quadrilateral has one pair of opposite sides parallel and one pair of opposite angles equal, then it is a parallelogram." This happens to be true. I think you will agree that it would be exceedingly difficult to recognize this as a converse of the original verbal statement of our theorem. It is interesting to note that we obtain precisely the same verbal statement for any of the following exchanges: H1 and C4, H2 and C3, H2 and C4. Of the 14 apparent converses possessed by this theorem only six are logically distinct after they have been verbalized. Of these six logically

[2] Nathan Lazar, "The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry," THE MATHEMATICS TEACHER, 31: 99–113, March 1938; 31: 156–174, April 1938; 31: 216–240, May 1938.

distinct converses four are true and two are false. This close knit system of theorems includes most of the basic theorems on parallelograms. Surely the generalizing effect of this procedure is evident. It is, moreover, a method of wide application. An analysis similar to the one shown above will reveal that all the theorems on congruent triangles belong to a mutually converse set and so may be headed by and derived from one key theorem. The same is true of all the theorems on similar triangles. These examples do not begin to exhaust the tremendous unifying effect of the multi-converse concept.

You will recall that there was a second part to our first question. Aside from its unifying effect, does the use of this method render the individual facts more significant? I believe that it does for two reasons:

In the first place, the only way a fact can acquire significance is in relation to other facts. Tracing these relationships increases the number of bonds linking this new fact to the student's previous knowledge. In this way, each new fact acquired by the student is properly oriented in the student's mind in much the same way that each detail of the landscape acquires its proper perspective in an aerial photograph.

In the second place, the student soon develops a proprietary interest in each new theorem which he "discovers" by using Lazar's simple definition. In life situations it is seldom required that we prove something which is already known to be true. There is usually real doubt in our minds about crucial questions. And there is doubt in the student's mind about each new statement which he encounters. He knows that the converse of a true theorem is not necessarily true. So he is asked to take each new theorem and state it in good English to be sure that he understands its meaning. Then he is asked to state his conclusion as to whether or not it is true. If he says that it is true, he is required to prove that it is true. If he says that it is false, he is asked to prove that it is false. This he may do, of course, by exhibiting a single exception. I believe that this method produces a more lasting impression than the conventional one where the student is told in effect: Here is a true theorem and you are required to learn the proof.

The second question: How can we develop an appreciation for the power and scope of a general statement?

I believe that one specific answer is to be found in the use of flexible diagrams. This, of course, at once brings to mind the ingenious flexible devices developed by Mr. John F. Schacht of Bexley H. S. in Columbus. Many of you are familiar with these devices and know their possibilities. I imagine that they would be very effective. However, the flexible diagrams to which I refer differ from Mr. Schacht's flexible devices in several important respects. They are really static drawings endowed with flexibility and elasticity by the imagination of the student. Since they are free of all mechanical limitations they may be deformed with abandon—that is they may be stretched without limit in any direction.

Let me try to illustrate this procedure. Consider this simple drawing:

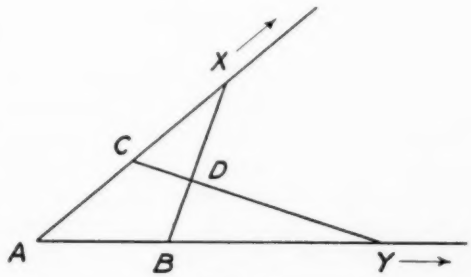

FIG. 2

We might ask: If A, B, C, and Y remain fixed while X recedes without limit along AC in the direction indicated, what does the figure $ABDC$ approach? If we allow X and Y to recede without limit at the same time in the indicated directions while A, B, and C remain fixed, what does the figure $ABCD$ approach? If $AB=AC$ and

X and Y move in such a way that $AX = AY$, then how does D move?

A second more complicated example is found in the harmonic construction:

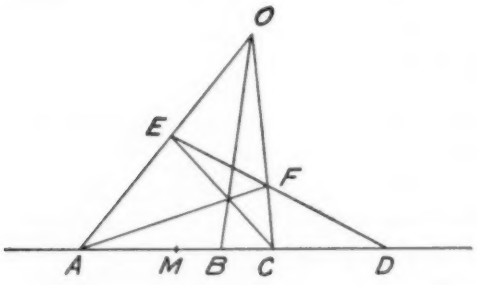

FIG. 3

An examination of this drawing reveals that if A and C remain fixed while B moves to the right toward C, then D will move to the left toward C and they arrive at C at the same time. As B approaches M the mid point of AC from the right, D acquires tremendous "velocity" to the right and eventually recedes to infinity—only to reappear an instant later coming in from infinity on the left as B passes through M. Of course, if A, B, and C are fixed the position of D is uniquely determined regardless of the position of point O provided it is not on AC. There are many other interesting facts about this drawing which we do not have the time to consider at the moment.

The need for endowing static diagrams with dynamic properties is often strikingly illustrated in later courses in mathematics. A certain trigonometry class was confronted with this question on a recent examination. "Referring to the drawing (fig. 4) how can you express x as a function of a, b, and θ?" Most of them gave the expected answer based on the law of cosines viz.: $x = \sqrt{a^2+b^2-2ab \cos \theta}$. Then they were asked: What is the value of x when $\theta = 0$? Remembering that the cosine of zero degrees is one, most of them wrote $x = \sqrt{a^2+b^2-2ab}$ and left the answer in that form. Now I'm not so much concerned about the fact that they failed to recognize the radicand as $(a-b)^2$. After all this is understandable when we consider that the cross product term appears, quite unfairly, on the end rather than in the middle. The thing that disturbs me is that it never occurred to these students to regard the diagram as flexible and let b rotate clockwise around C as a pivot and so literally watch x take the obvious value $a-b$ when $\theta = 0$. In this instance, failure to use the flexible diagram technique has resulted in failure to appreciate the generality of the relation stated in the cosine law

These examples are, perhaps, sufficient to illustrate the use of the so-called flexible

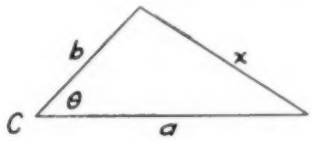

FIG. 4

diagram. Students quickly acquire the ability to deform diagrams in this fashion and they thoroughly enjoy the process. It appeals to their imagination by rendering geometry dynamic. After they have had some experience using this method we can begin to point out to them that even when these drawings are subjected to extreme distortions, certain relationships remain true. This is a manifestation on the elementary level of what Klein meant when he said, "Geometry is the study of the invariants of a configuration under a group of transformations."

Let us pause briefly to consider a very simple example of this type of invariant relationship. Suppose we have four points, A, B, C, and D on a straight line. Euler's

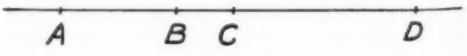

FIG. 5

relation states that $AB \cdot CD + BC \cdot AD = AC \cdot BD$. This relation remains true no matter how we change the lengths of these segments. It remains true even if we permute the four given points, provided we allow the use of directed line segments.

Now having illustrated the flexible diagrams and noted the significance of invariant properties, let us turn our attention to a specific answer for our second question.

Consider the theorem of Apollonius which states that the sum of the squares of two sides of a triangle is equal to one-half the square of the third side plus twice the square of the median to that side.

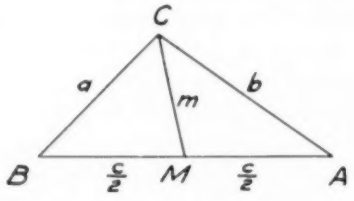

FIG. 6

For the above diagram we have: $a^2+b^2=(c^2/2)+2m^2$. This relation is supposed to be the invariant property of this diagram no matter how it is distorted or deformed so long as M remains the mid-point of BA. The question is how do we proceed to develop an appreciation of these generalizations in the student's mind?

First allow me to solve the above relation for m^2. We have

$$m^2=\frac{2a^2+2b^2-c^2}{4}.$$

Now we can ask a number of questions designed to reveal this theorem as a unifying principle linking together a number of heretofore unrelated geometric facts. We will keep the points A, M, and B fixed but we will regard a, m, and b as elastic so that the point C may move freely in the plane. Suppose we allow C to coincide with B. Then it is obvious from the drawing that m must take the value $c/2$. The question at once arises as to whether the formula is valid for this situation. In order to test the formula we must substitute $a=0$ and $b=c$. We obtain at once $m=c/2$ and observe with satisfaction that the formula is still valid for our degenerate triangle.

Now suppose that C moves in such a way that $a=b$. Then the triangles BMC and AMC become right triangles and according to the Pythagorean theorem we should have $m^2=b^2-(c^2/4)$. To test our formula in this case we substitute b for a and quickly obtain the anticipated value. Next let C become the third vertex of an equilateral triangle. In this case m becomes the altitude of this triangle and we know that it should equal $c\sqrt{3}/2$. Again our formula gives the correct result when we substitute c for a and b. Finally, let point C move on a circle having AB as a diameter. Then the triangle ABC has a right angle at C and we know from a previous theorem that the median to the hypotenuse of a right triangle is equal to one-half the hypotenuse. Our formula is again vindicated when we substitute c^2 for a^2+b^2. In each of these cases we determined what our formula should become from the geometry of the drawing, and then we found that we obtained the same result from the algebra when appropriate substitutions were made. In this way, a resonance is developed between the geometry and the algebra and each renders the other more meaningful.

I believe that this kind of a discussion is of some value in developing an appreciation of the power and scope of a general statement. Once the appreciation is acquired, the students begin to use each new generalization as a vantage point from which to survey surrounding territory. Having found and appreciated one generalization, they may want to find others. Someone may ask if there is not some other theorem even more general than the theorem of Apollonius. The answer is "yes." Our theorem is a special case of an even more general theorem known as Stewart's Theorem. The class may not consider it but at least they have had a glimpse of that inspiring vista leading to increasing generality.

This brings us to our last question: Is there a principle which can be used for the purpose of deriving generalizations in geometry? As a partial answer to this

question, I offer the principle of continuity in approximately the form stated by Poncelet in 1822. This principle asserts that a property which is once demonstrated for a figure in one of its general forms remains true as the figure is changed continuously in accord with the conditions under which the property was first demonstrated and will be true when the figure takes a limiting form. It is not necessary to state this principle explicitly for the edification of high school students but it, like the concept of dependence, should be a dominant idea in the teacher's thinking. Indeed, when the principle of continuity is implemented by the use of flexible diagrams, we have an effective combination for the study of continuous change. We obtain our generalizations by trying to find relations which remain true as certain changes take place.

For example, consider a circle and a secant drawn from an external point as shown in the diagram. We ask what the product $PX \cdot PY$ depends on as PXY rotates on P as a pivot. We know that this product is equal to $\overline{PT}^2$ where PT is the

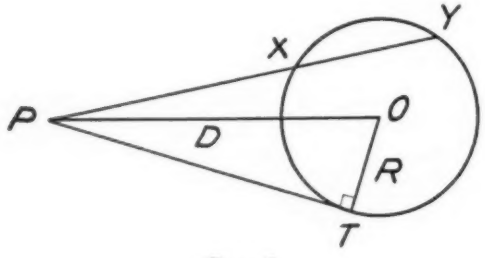

FIG. 7

tangent or the limiting position of the secant. Now we ask what does $\overline{PT}^2$ depend on? The student can see that the length of PT depends on how far the point P is from the center of the circle and also on the length of the radius R. From the right triangle PTO we have: $\overline{PT}^2 = D^2 - R^2$, hence $PX \cdot PY = D^2 - R^2$. Now the question is: Does this relation remain true as the drawing is subjected to continuous change? Let us define the product $PX \cdot PY$ as the power of the point P with respect to the circle. We see then that if the point P is outside the circle, this power is positive as we would expect it to be since PX and PY are then measured in the same direction. If P is inside the circle our product is negative for then PX and PY are measured in opposite directions and, of course, in this case the quantity $D^2 - R^2$ would also be negative. As a matter of fact, the above relation is true for any position P may take in the entire plane. We have then the general theorem: "The power of a point with respect to a circle is equal to the square of its distance from the center of the circle diminished by the square of the radius of the circle." Hence this product remains constant as long as R and D remain fixed and so we have one theorem equivalent to the three theorems commonly stated relative to intersecting secants, chords, and tangents and a circle.

Another example of the usefulness of the principle of continuity for the purpose of obtaining generalizations is found in the statement: "The angle included between two lines of unlimited length which meet a circle is measured by one-half the algebraic sum of the intercepted arcs." This statement is true if we agree to regard concave arcs as positive and convex arcs as negative. Having adopted this convention we have a perfectly general statement which includes all the theorems concerning the angles formed between two chords, two secants, etc. A little reflection will disclose that this principle of continuity may be utilized to obtain many other generalizations.

I close this discussion by citing one more example. In most plane geometry texts we find a theorem known as Ptolemy's Theorem. This theorem states that if A, B, C, and D are any four points on a circle then we have the relation: $AB \cdot CD + BC \cdot AD = AC \cdot BD$.

Suppose we allow this figure to change so that AC and BD become diameters. Then all the angles of the quadrilateral become right angles and the above rela-

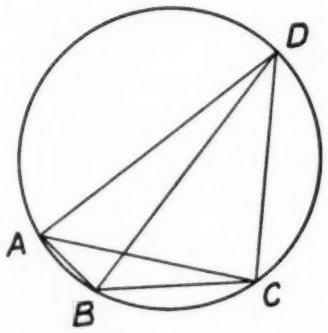

FIG. 8

tion becomes the Pythagorean theorem.

On the other hand, suppose we allow the radius of our circle to increase without bound while the length of the arc *ABCD* remains fixed. Then of course the arc approaches a straight line and according to Poncelet's principle of continuity, the above relation becomes Euler's relation for four points on a line which we considered previously.

In conclusion, then, we have:

1. The *multiconverse concept* for the purpose of tracing the relations between geometric facts.
2. The *flexible diagram technique* for the purpose of developing an appreciation of the power and scope of a general statement.
3. The *principle of continuity* for the purpose of deriving generalizations.

These are, of course, only partial answers and I do not know the extent to which they are acceptable. I only know that I have used them for a good many years and they seem to be effective. At any rate, I hope that they are definite enough to serve as a basis for discussion and thereby focus attention on the important problem of teaching for generalization.

If we can demonstrate that teaching for generalization is possible, surely this will strengthen our contention that teaching for transfer is possible. No one questions the utilitarian values of geometry but many educators contend that these utilitarian values alone are not sufficient to justify geometry's position in the curriculum. These criticisms subsided during the war for then mathematics, like guns and airplanes, was an instrument of national survival. Then the utilitarian values of mathematics were overpoweringly evident. But with the passing of the crisis, the old criticisms returned to jeopardize the position which geometry now occupies and with it the entire structure of secondary mathematics. The present position of mathematics is not unlike that of the British Tommy in Kipling's famous poem.

> "For it's Tommy here and Tommy there,
> and Tommy, wait outside;
> But it's special train for Atkins when the
> troopers on the tide"

Let us not waste time in surveying this situation ruefully. Nor can we question the sincerity of our critics. Instead, those of us who teach geometry and therefore believe that it possesses unique and irreplaceable values, must speak up now with vigor and conviction in order to preserve this heritage for all the boys and girls in America who are capable of profiting by it.

Copies of the First Yearbook Available

A limited number of fresh copies of *A General Survey of Progress in the Last Twenty-five Years, The First Yearbook of the National Council of Teachers of Mathematics* were included in the library of Raleigh Schorling who was the book's editor. Mrs. Schorling has offered to make these available to libraries and individuals who wish to complete their files.

Copies may be obtained for $2.50, postpaid from Phillip S. Jones, Mathematics Department, University of Michigan, Ann Arbor, Michigan. Mailing and billing will be simplified if a check or money order for the full amount is included with your order.

Observations on Attitudes of Young Children Toward Mathematics

By Emma N. Plank

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I. Introduction

A study of the development of concepts of number was undertaken to throw light on the learning processes involved. The materials used were mainly those devised by Montessori. The twenty children who served as subjects ranged from kindergarten to the sixth grade. In order to study the factors contributing to failure in arithmetic and to find out whether they are specific and different from causes of failures in other fields, children were chosen who were successful in all other school subjects. It was also hoped to see whether different teaching methods might resolve the difficulty.

It would have been desirable to select as subjects children who had been studied in one of the guidance clinics because information concerning the child and his family would have been available. A preliminary survey brought the first surprise: none of the three clinics approached dealt with cases specially marked by difficulties in arithmetic. Since children in hospital clinics were referred by pediatricians their problems were of a general nature, especially concerning the mother-child relationship. The Guidance Clinic of the Board of Education was swamped with referrals from schools; but where difficulty in learning was the reason given, it was reading not arithmetic. Only in junior high school and high school did arithmetic play a role in the referral; but these cases were outside the scope of study.

At the same time, remarks like this are found in the literature: "Arithmetic is the principal cause of pupil failure in the middle grades" (17). Or: "More pupils have been required to repeat work of grades above the second because of failure to master the arithmetic process"(4).

As there were no children available for study through the child guidance clinics, an elementary school was selected. Classroom teachers from the third to the sixth grade were asked to select children with specific difficulty in arithmetic for the study. In addition, a few children from the school where the writer had previously taught who showed very good ability in arithmetic were included in the study.

The public school which was chosen is an interesting school in a midtown area. Some of its children are transients, living in hotels with their parents until permanent quarters can be found; others live in rather small and crowded apartments.

Twenty children were selected for individual study. They included:

1. Children who had not gone far enough in school to have received formal instruction in number work.
2. Children with special facility for arithmetic.
3. Children who had some difficulty with arithmetic.

The conditions for research turned out to be far from ideal. The school was not set up for it, and the idea was new to the parents. Some parents did not respond when their consent was sought. There was no chance to expose the children exclusively to the new approach or to exclude conditions which we thought detrimental to the child's growth in arithmetic.

For these reasons, the writer's interest turned from the observation of the value of materials as teaching devices towards factors in the personality structure of the different children, and their previous mathematical experience which perhaps

accounted for their difficulties.

The results of the investigation are valuable mainly in that they show a direction for further study under more favorable conditions. Several observations seem to lend themselves to intensive followup. They will be further discussed in the conclusions of this paper.

II. Review of Literature

Though a vast literature can be found on personality factors in relation to learning how to read, very little was available on relations between personality and success or failure in arithmetic. Several authorities in the field who were consulted agreed that very little was known and encouraged study in this direction.

Drummond's remark, written twenty-five years ago, is significant:

> General ability is not always attended by mathematical ability. . . . If these sensitive children are hurried or puzzled or allowed to feel that they have made a fool of themselves, they will give up number and find themselves in some other sphere. . . . The difficulty and its solution are to be sought in the region of emotions (6).

Hildreth found less relation between intelligence and achievement in arithmetic than in either reading or spelling, thereby also pointing to an emotional component (11). On the other hand it is known that children successful in arithmetic have high I.Q.'s Our case studies also showed this.

In studies from the psychoanalytic point of view, descriptions of severe restrictions of ego activities and inability or refusal to take part in class activities point again to an emotional component. Aichhorn's case study of the child who could not learn how to multiply shows that the horror unconsciously connected with the number eight was the obstacle that had to be removed (2). Just-Kéri reports a case where the number symbol stood for a real happening in the child's life and where the appearance of this number brought about day dreams in connection with this experience instead of a response to the problem. (It was the number five that reminded the child of her father's leaving the family when she was five years old) (13).

Adler mentions the fact that people with strong feelings of insecurity are usually bad in arithmetic; on the other hand "no field of knowledge gives a greater sense of security than mathematics" (1).

Similar considerations are found to have been integrated into the rationale of the digit span and arithmetic subtests of the Wechsler-Bellevue intelligence test (18).

Levy's study on the relation of maternal overprotection to scholastic performance was of particular interest to the writer. Levy brought out the relationship between achievement in reading and in arithmetic. Table I, adapted from page 96 of his book, shows the significant differences in the achievements of different types of children (14).

Table I*

Reading Arithmetic Retardation Ratio

Group of children studied and number of children in the group	Percentage of children retarded one year or more in:	
	Reading	Arithmetic
Pure overprotected cases (8)	0.0	37.5
Mixed overprotected cases (18)	5.6	22.2
Rejected group (14)	28.6	21.4
Check group (369)	23.8	29.8
Special School group (150)	12.0	39.6

Fernald includes an account of her technique in remedial work in arithmetic in her book, but does not offer as original an approach as in her work on reading (9).

Very few other studies exist. In an early paper (1921) failure in arithmetic is traced to school absences and defects in the educational process rather than in the child's innate mental makeup (19). A series of papers on remedial cases in arithmetic was published under Brownell's guidance (3, 8, 10, 21). Some of these

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children failed because they disliked speed tests. They all seemed to lack confidence and were easily discouraged.

One is not astonished to find these observations if one considers attitudes of teachers as expressed by Sweeney:

> Children should master certain fundamental number relations before they are permitted to advance . . . nothing else than hundred per cent accuracy is desirable. . . . Fortunately this statement of drill does not entail additional work for the instructor. At the contrary it relieves the teacher of much work in preparation for classes and progress of the children can be observed (20).

This method is supposed to prevent errors and make remedial work unnecessary. We wonder how rigid children, who as a group are likely to have difficulty in arithmetic, react to a declaration of rigidity on the teacher's part.

Buswell mentions the high percentage of failures in arithmetic as a reason for general dislike of school which is detrimental to personal adjustment. If organized subject matter can be arranged to be stimulating rather than depressing, it contributes to personality development. The overemphasis on speed in computation may produce a feeling of dullness in quantitative situations (5).

Hildreth remarks on the feeling of continual failure which makes arithmetic so distasteful to a large number of children (12).

Turning now to the twenty children who were the subjects of this study: What insight into their personality structure and reasons for failure or success may be gained from the observations?

III. The Children: Evaluation of Their Background

The twenty children observed can be grouped into three categories:

1. Three average young children, 5.2, 6.2, and 7.1 yrs. respectively, two boys and one girl, who had not had formal instruction yet. They attended kindergarten, first grade, and second grade, and were neighborhood friends of the writer's.

2. Four children ranging in age from 8.8 to 10.5, all boys, somewhat accelerated in arithmetic. One was in the third, two in the fifth, and one in the sixth grade. They were selected from a group the writer had taught several years ago; they were slightly familiar with the material used.

3. Thirteen children ranging in age from 8.0 to 11.8, seven girls and six boys, somewhat retarded in arithmetic. Eight attended the third, one the fourth, one the fifth, and three the sixth grade.

The I. Q. of the young group ranged from 110 to 121; of the accelerated group from 126 to 154 with a mean of 139; and of the retarded group from 90 to 126 with a mean of 109.

The level of socio-economic background of the children in the retarded group as expressed by their father's occupation was fairly evenly distributed from unskilled workers to highly specialized professional men. The young group and the accelerated group had safe economic backgrounds. Four of the seven children in these two groups grew up without their fathers living with them regularly.

The school grouping of the children ranged from kindergarten through the sixth grade. None of the children had been kept behind on account of their arithmetic retardation, since they were all good readers. The only exception was Jack F. (age 8.11)[1] who entered school at the age of seven Two of the accelerated children were a year ahead in their grade placement.

The retardation expressed in a deviation from the norm as measured by standardized tests ranged from 3 to 14 months (one school year counts for ten months). None of these children had any difficulty with reading; some were avid readers.

The acceleration ranged from 4 months to over 2 years.

Changing of school played a role with the retarded group. Of the 13 children, two had one change of school, three had two changes, and one had three changes of school.

The school attendance was fairly regular

[1] Names disguised. Ages as of October, 1946.

for most children. Minor illnesses kept two accelerated and two retarded children home at frequent intervals, thereby interrupting the continuity of their schooling.

The position of the retarded children in their family group seems of particular interest:

5 were "only children."

6 were the youngest in the family, the next sibling being in the last year of high school, at college, working, or living away from home and married.

The remaining 2 are five years older than their next sibling; both have very exacting mothers who express constant dissatisfaction with them.

Not one of them had siblings as close in age as usual.

Parallel with that, eight of these children had parents considerably older than the average (some between 50 and 65 years old).

In seven families at least one parent was foreign-born.

One of the children lives separated from her parents on account of divorce. One father is dead.

In the young and accelerated groups no special position in the family group could be found. One is an only child, two the oldest, two the middle, and two the youngest in their family.

The information on early developmental history given by the parents was too vague to allow tabulation.

IV. The Setup for the Observations

Information regarding developmental, family, and school history of the children was secured through school records and through interviews with the parents, mainly at home visits. Most of it was assembled prior to the initial interview with the child.

One hour was scheduled for the initial interview which took place in an unoccupied classroom or in a room adapted in the writer's home. The children were allowed to go on beyond the hour if they were still absorbed in their work.

A full set of the Montessori mathematical material for elementary schools was set up in the observation room. This material has been developed by Maria Montessori to offer young children a chance for individual experimentation on quantitative and spatial relationships. It includes very interesting materials for the clarification of the decimal system. Details on the material can be found in books by Maria Montessori (15, 16) and in a recent pamphlet by Margaret Drummond (7) which gives an excellent introduction.

During this initial hour, rapport with the child was established, and the children selected parts of the material for their work and became acquainted with its use.

From this group of twenty children, four third-graders and one fourth-grader who was particularly retarded, were chosen for a weekly followup through four months. Three of the third-graders worked during the same sessions. The intention was to approximate a more natural situation. The two other children worked on an individual basis as they seemed to require more personal attention. Throughout the observation period new material was suggested by the writer only if the child seemed at a loss what to do next.

To supplement the scores of intelligence and achievement tests which were part of the school records, the writer gave the following tests to these five children: 1. an arithmetic concept formation test;[2] 2. the arithmetic part of the Progressive Achievement Test; 3. the Hanfmann-Kasanin Concept Formation Test;[3] 4.

[2] The test involves the use of twenty-two blocks in five different colors, six different shapes, two different heights, and two different widths. The subject is to discover how these twenty-two blocks can be divided into four kinds. The solution is the division into tall-wide, tall-narrow, low-wide, and low-narrow groups (18, p. 209). The test explores the way the subject reacts to concepts of a quantitative and spatial nature when faced with a baffling problem. Such qualities as fluidity, flexibility, rigidity, and persistence can be noted, as will be shown in discussing the performance of Jack F.

[3] Test on Arithmetic Meanings and Vocabulary for Grades Two and Three, copyright 1937 by Scott, Foresman, & Co.

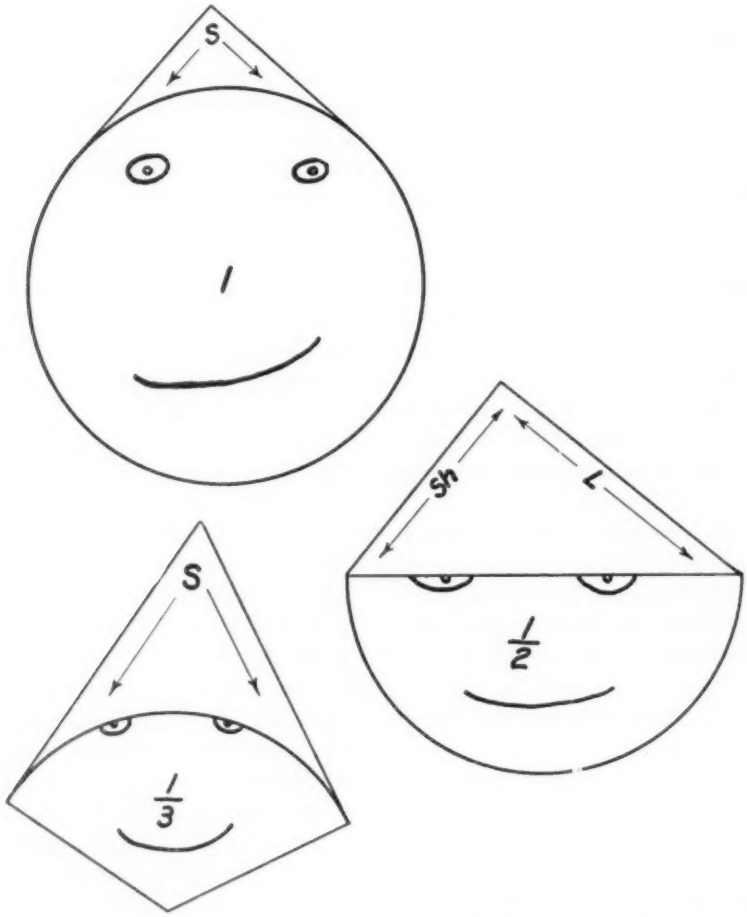

FIG. 1. Copy of imaginative work with the fraction material.

the Rorschach Test, for two children whose behavior seemed puzzling.

V. OBSERVATIONS ON THE WORK WITH MATERIAL

During the work periods with the material the following observations could be made:

All children of the study took to the material with great interest. The accelerated group showed greater perseverance and originality in its use which may have been the result of their greater familiarity with it through previous use, two and three years ago. The time spent in work during one session with the material ranged from 35 minutes to 85 minutes. Usually the children would work with two different materials at one session.

The difference in the spontaneous use of a set of circles—material used to clarify fractions—by two boys of the same age (8.7) shows what varied and creative use can be made of it. Paul N. (slightly accelerated) worked with the fraction material shortly after Halloween. In cutting out his whole circle he noticed that the corner of the paper left on it looked like a hat. He proceeded to make a pumpkin face of the circle. The arrows and the S on the "hat" indicate that the sides of the triangle have the same length. This shows how alert he was to relationships of quantities and that he did not just play. The result pleased him and he went on trying the same with the half and all through the sixth of a circle. The sh and L on the half indicate that one side of the triangle was

short and the other long (Fig. 1).

Norman S. (slightly retarded in arithmetic, with excellent abilities in every other field) went about it in a methodical way. He was interested in the small differences in size between an eighth, a ninth, and a tenth. After having traced the whole circle and carefully inserted eight eighths, he showed the differences in the size of an eighth, a ninth, and a tenth, by drawing them inside each other and separately (Fig. 2).

Norman is a child who has to achieve tangible results in whatever he does, for his own feeling of satisfaction and to please his exacting mother, while Paul goes about his work in an unpreoccupied, therefore more creative way.

Some of the remarks the children made while working may help to illustrate their relationship to arithmetic and the material.

The accelerated group talked very little and worked with great self-direction and independence.

Walter L. (10.5:) You get tired of arithmetic unless you use it for something real, like scale models. . . . Doing beads, and multiplying square feet, you have first to do it to understand it, then you keep on practicing. It is more fun to do your arithmetic than to copy from the black board. (He is an astigmatic child who had great initial difficulties with reading before his defect was discovered.)

Paul N. (8.7): I found out how to multiply big numbers! (He brings the short chain from the bead-board): Look, 8+8=16, 16+16=32, this is 4×8, 32+32=64, this is 8×8.

In the retarded group's remarks emotional components became evident. Some children express relief that the material helped them:

Jack F. (8.11): You can count easier with beads, otherwise you have just a handful of something. (While working with the cards to build large numbers): These are life savers! (Coming home to mother, after having worked with the bead-board): Mother, did you know that there are ten tens in a hundred?

Susan L. (8.10): Arithmetic is much easier for me with your things, maybe I can concentrate a little more.

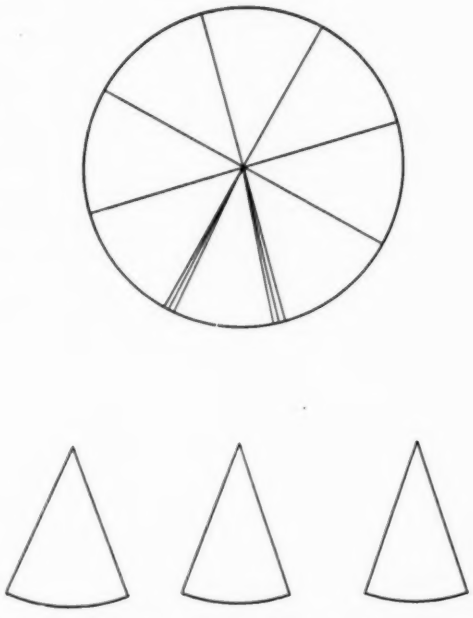

FIG. 2. Copy of methodical work with the fraction material.

Others express their anxiety:

Robert A. (9.1) (talks constantly during the first fifteen minutes of his initial interview): I get mixed up . . . I don't know what that (multiplication mark) is . . . I can't ask the teacher, she is too busy . . . I don't know what 8+4 is . . .

Carolyn C. (10.5): I like to do arithmetic, but when I get stuck I get nervous and I forget the number I'm working with.

Some children offer indications for their discouragement:

Brigid G. (11.7): I usually don't like arithmetic because I'm not so very good in it, but this is fun, I like it (while multiplying with the bank game).

Norman S. (8.7): I don't like arithmetic because I don't like to miss problems, but I always do. (As he works filling in multiplication blanks): You don't mind if I keep this blank for my mother? (After having filled in all multiplication blanks from one to ten, he starts making one himself for the combinations of twelve): Don't think that I am nuts. (Claps his hands in enjoyment over his success): Well, this isn't so bad as I

thought. (After his session of a full hour is over and the next child arrives): May I keep on working, I won't disturb you.

VI. Followup Study on Five Children

The four third-graders and the fourth-grader, who were followed up through four months, showed a variety of reasons for their difficulty with arithmetic. The children were not selected on this basis, however. They were chosen for the follow-up because it was desired to have a group of children of the same age, and the eight-year-olds happened to represent the largest group. It was also hoped to gain more insight into the development of the difficulties in arithmetic by studying younger children. The chance for developing a healthier attitude towards arithmetic seemed greater, if the feeling of failure and the lack of interest had not yet become too deeply implanted in the child's mind.

Table II

The Children of the Followup Study

Name	Sex	Age	Siblings	I.Q.	School changes	Retardation in months	Percentile rank in arithmetic meaning test	Hanfmann-Kasanin Test
1. Carol B.	F	8.10	none	118	none	3	80	flexibility
2. Jack F.	M	8.11	bro., 17	108	late start	6	40–50	fluidity, anxiety
3. Dick K.	M	8.0	none	100	2	6	50	fluidity
4. Susan L.	F	8.10	none	108	3	3	30–40	flexibility
5. Robert T.	M	9.1	five grown up	110	none	12	not given	rigidity, insecurity

These children were: (Table II)

Case 1, Carol B.: Carol is a shy, rather reticent little girl. She has slight strabismus which is not corrected at this time. Carol is a lonesome child. Her father, an older man, is at sea with the Merchant Marine most of the time. He is divorced from her mother, but intends to remarry her in the nearest future. The mother has many outside interests; she is rarely home when Carol comes from school. It was easy for for Carol to withdraw into a dream world with her books. Her teacher was not the person to break through this detachment and to make reality interesting to the child. Carol is a perfectionist who will attack a problem only if she is sure of success.

After the initial interview Carol became quite friendly with the writer and did not present any difficulty in her work with her. In the middle of the observation Carol was promoted to another grade. Her new teacher describes her now as one of her good students in arithmetic. There was no lack in comprehension, as the table shows; Carol's attitude accounted for the retardation. She is a child who does not like to be rushed. Evidently all Carol needed to try a field in which she was not superior from the start, as in reading, was some encouragement and somebody to take personal interest in her. Her original teacher was a stern, over-competent person who probably made Carol withdraw further; since the child acquired more assurance through her work with the writer, the warm interest which her new teacher showed in her children provided a good start. The personal attention that could be offered her in the small group, and a change of teacher personality cleared up the difficulty for the moment.

Case 2, Jack F.: Jack has been overprotected since his birth as a premature baby. He comes from a family which is comfortably off, with a loving and devoted mother, a busy father who travels a great deal, and a seventeen-year-old brother. He came from a Pacific island after Pearl Harbor and has lived in three different

states since. He was sick a great deal between the ages of four and seven and started to go to school only after he came to San Francisco in the late fall of 1944.

Jack is an extremely sensitive child, restless, with great difficulty in coordinating his small muscles. Except for his vocabulary, his interests and behavior were on a much younger level when he started school. His Stanford-Binet test yielded an I.Q. of 90. Thirteen months later his test was repeated and he scored 108. He has learned to read and spell adequately in spite of his extreme lack of concentration, but was unusually confused in his understanding of quantitative relations. This confusion also showed in a lack of understanding of interrelation of the world around him and was also evident in any group discussion. This has been particularly noticeable as he has an outstanding vocabulary.

His immaturity was equally marked in his paintings and drawings; he started out on the four-year-old level though he was seven. His emotional immaturity was evidenced by unexpected remarks and interests in toilet procedures which were on the same young level. It seemed as if he had to relive a phase of his development which he had missed out on before he was ready to start learning. The writer wonders whether Jack may be one of those children whose early sexual interests and investigations had been discouraged and interrupted and who since "did not know," "could not find out," in other words, was not able to function adequately in other intellectual areas where finding out for onself and drawing conclusions are essential.

The first thing for which Jack achieved approval from his contemporaries, in his second year at school, was in making up and telling highly imaginative stories. His extreme sensitivity for music became apparent. The rise of 18 points in his I.Q. within a year also points toward an emotional blocking of his intelligence at the beginning of school. Insecurity and anxiety are still basic patterns of his personality and are apparent in the Rorschach and Hanfmann-Kasanin tests. But in general Jack can now function with a group of his contemporaries.

A tabulation of the Rorschach results[4] and a blind interpretation[5] follow:

W	12+(3)	M	3	H	3
D	(3)	FM	4+(2)	A	7+(3)
S	(1)	m	(1)	Aobj	1+(1)
	—	K	(2)	Obj	(4)
	12	F	5+(5)	N	(3)
		Fc	(1)	Geo	1
			—	Smoke	(2)
P	4+(1)		12	Abstr	1
O	(1)				—
					12

"J. is without question an anxious child. There is lack of free affective dealing with others—it is to be assumed that he does not get along well with other children. Probably his inner impulses overwhelm him so that he cannot establish friendly contacts with other children.

"The high F%, however, indicates that he inhibits, or rather tries to inhibit, the expression of that amount of aggression; but this is not successful.

"There are no signs of psychosis, or hardly any. The wolf's head in card I is a little suspect, and there is a great deal of fighting and escaping. There is evidence of lots of intellectual drive, but as far as we can see he can not live up to it."

Jack's work with the Hanfmann-Kasanin test[6] was of particular interest. The record of his performance follows:

Jack started out with the remark: "They look like a graveyard" (the blocks representing tomb stones). He began his work by counting off four orange blocks and putting them in one corner. He then put arbitrary groups of four blocks each into the three other corners. When told that six blocks were left, he tried to group all blocks into four groups by color.

Noticing the impossibility, he moved all blocks back into the center and began moving single blocks without a visible system. Asked why, he replied: "I want to make some sort of a picture." He proceeded to build towers of several blocks.

At this point the examiner turned over a few blocks and showed Jack the names. Upon discovering two with the same name, Jack tried to build "sort of a house-looking shape" with them, and from this point on attempted to form patterns of blocks bearing the same name, ascertaining the names by frequent turning.

[4] Scoring symbols according to Klopfer and Kelley.

[5] Mr. Allen T. Dittman, clinical psychologist, was kind enough to interpret Jack F.'s Rorschach.

[6] See the description of the test, above (p. 256).

Four groups were formed thus, leaving nine blocks in the center. Encouraged by the examiner, he began to move those singly—but arbitrarily, waiting for the examiner to turn them over and to show him whether the move was right.

In this fashion the grouping was completed after 19 minutes. Jack failed, however, to recognize the underlying principle. Shown the difference in height of blocks, he could see that, but shown the difference in size, he could not.

Marked fluidity is evidenced by the lack of planning, the hit-and-miss trying, the repetition of mistakes, and the jumpiness.

For a child like Jack, the Montessori material is of unusual value. He can find relationships for himself, he is urged to manipulate and investigate. He does not face competition or domination while he works. His motor restlessness can be absorbed by many manipulations. His remark, "You can count easier with beads, otherwise you just get a handful," shows how important the clear organization of material is for an insecure and confused child.

Jack scored six months' retardation when he took the Progressive Achievement Test in November, 1946, in his school. At the end of March, 1947, when the writer retested him, he had gained nine months. He now scores three months above his grade norm as far as the total score is concerned. The scores on the subtests though, are widely scattered.

Case 3. Dick K.: Dick is both physically and emotionally immature. He is an unusually small and pale little fellow and shows extraordinary motor restlessness. His home environment seems to be quite problematic. His father, a veteran, is not working regularly and is mostly at home (with no apparent physical disability) while the mother supports the family. Thus the roles of the parents seem reversed. While Dick gives the impression of a physically neglected child, his father on the other hand overstimulates him intellectually and fosters "Quiz Kid" responses. Before Dick entered the public school he went to a parochial school in the Midwest; the father said that there was no difficulty there and implies that the new school might be at fault.

Dick's main difficulty at school is that he cannot apply himself. Since he is a fluent reader this showed first in his work in arithmetic, but it gradually became apparent in every field. He smears all over his papers and completes assignments only if his parents do them with him. Then he brings immaculate work to school, far beyond his standards. At school he hardly completes a task; he has to be held down constantly and even then can not organize his work. His anxiety is evident but is usually covered up by quite assertive behavior. Unfortunately, he is by far the smallest child in his room and as such and as a newcomer to the city has to assert himself constantly with the other children. He craves attention from his teacher and often wants to stay after school to work with her. He is then affectionate and can finish his work under her direct supervision, but as soon as he is back in the group situation the old pattern prevails.

Dick profited least of all the children by the work with the writer, as far as better adjustment in his classroom is concerned. He enjoyed working with the material but this did not change his attitude in class. When he had finished work with some material, he needed immediate attention to choose the next and settle down again. If this was not done he would attract attention through negative behavior. The change of schools—from parochial to public school—coincided with the father's return from the Army. Unfortunately the father's position in the family at present does not facilitate the adjustment of his son.

His teacher and the writer concluded that the child's difficulties had not changed and that intensive guidance work with the parents and the child seemed advisable. It appeared that rather intimate family problems would have to be discussed, which were outside the scope of this study. Dick will be referred to a child guidance clinic if his parents want to cooperate.

Case 4, Susan L.: Susan presents a quite

different picture. She lives separated from her divorced parents; she stays with her paternal grandmother and an aunt in a hotel room. The father's family are Syrian-Jewish merchants, while the mother is Catholic and works as a waitress in another city. The father is about to join Susan in San Francisco.

The very unstable and often shifting family situation finds an expression in the frequent changes of school. Susan went to school in three different states. Thereby the continuity which is so important for the formation of mathematical concepts never existed.

Susan is an attractive little girl. She craves affection, but her restlessness and easy distractability bring her more nagging than affection in the classroom. This was mitigated by her work with the writer. Susan profited visibly by working with the material. Her understanding could be broadened and some gaps filled. For instance, she could not count beyond a hundred, but learned it quickly after she got interested in the chains of the beadboard. She developed better concentration and is getting along all right in her classroom right now.

Case 5, Robert T.: Older than the other children and in the fourth grade, Robert was seen occasionally only. He attracted the writer's interest as he talked incessantly during his first interview and showed deep concern over his inability to achieve anything in arithmetic.

Though he spoke poor English when he began school (his mother came from the Philippines), he is very good in reading and spelling. His failure in arithmetic, though, was dismal. He did not score a single correct answer in the Arithmetic Fundamentals Survey Test administered at the beginning of the fourth grade. He is the youngest child in the family, his five siblings are married and live away from home. His parents are elderly, hard-working people. The family lives in a basement.

Robert is very interested in mechanical matters. He has built himself a coaster and a raft. He mentions that he likes to tear apart machines and that he dislikes puzzles. A Rorschach test seemed indicated. Results (scores and interpretation)[7] follow:

W	8+(4)	M	1+(1)	H	1
D	6+(10)	FM	4+(5)	Hd	1
S	1	m	(1)	A	5+(7)
	—	F	7+(4)	Ad	(2)
	15	Fc	2	Aobj	2+(1)
		FC etc.	1+(6)	Obj	2+(2)
			—	Art	1+(1)
P	7+(1)		15	N	1
				Pl	1+(1)
				blood	(1)
				mask	(1)
				abstr	1
					—
					15

"The most striking feature in this record is hyperactivity. The child is overresponsive to environmental stimuli; yet he does not seem to use them to good advantage.

"He seems to be aware of his surroundings, enters into activities with others. He has, however, raised a barrier of restrictions, probably as a defensive device; the F% is significant in this respect.

"In a good many ways Robert is infantile, especially in his dealings with others. All the eating that occurs in the colored cards is infantile; it seems to be aggressive eating, possibly indicating hostility towards the mother—the mother who is inaccessible emotionally.

"Robert is aware, perhaps over-aware, of the social norms of his group. This awareness may be a form of escape: if he can be like the others, he is protected. Yet there is some indication of negativism. He is not overly intelligent. He has intellectual drive, but not too much of it; but he can not measure up to it. There are no signs of psychosis."

The writer retested Robert with the Progressive Achievement Test in November, 1946. He scored a grade equivalent of 3.1 (12 months retardation). In January, 1947, he scored nine correct answers on the test which he had failed completely in the Fall; three weeks later, in a routine test in his room, he tested only eight months behind his actual grade. His teacher, a very understanding person, felt that he had improved very markedly; she thought the progress was due to the personal attention which could be given him in individual sessions. The writer doubts that the help given will be of permanent value, but is sure that this

[7] See footnotes above (p. 259).

child would profit greatly from consistent use of the individual approach.

VII. Summary and Conclusions

The observations of this case study can be grouped as answers to these two questions:

1. Did work with the Montessori material help these children in acquiring meaningful number experiences?

2. Was there any insight offered into the personality structure of these children expressed in their relation to arithmetic?

Question 1: In the new setup provided by the facilities of the observation group no difficulties in concept formation could be observed. The children frequently undertook problems involving concepts which they were not familiar with from their classroom work, and showed a quick grasp. For instance, the retarded third-graders successfully worked with addition and subtraction of fractions and added or subtracted five-digit numbers. A ten-year-old in the advanced group chose to work on the theorem of Pythagoras and found the principle of its application to both the isosceles and other right triangles by his work with the material.

A good span of interest and great perseverance could be observed. Manipulation of the material allowed for longer periods of concentrated work than would usually be planned for and asked of young children. The children worked 20 minutes on the average, with one material, while they spent as much as 50 minutes in investigating a problem in which they were interested. The writer found, as she did also previously in many years of classroom teaching, that the material was equally stimulating to the young as well as the accelerated or retarded group. Work with materials increased the understanding and enjoyment of all the children and challenged them to further investigation. Since the "number environment" of the children's classrooms was found inadequate for fostering a relationship to quantitative thinking, the observation had its value in providing just that.

The material established the rapport with the children, thereby offering some insight into the interaction of the children's personalities and their learning in arithmetic. It relaxed and challenged the children's thinking at the same time.

Question 2: Though in the retarded group various reasons for the individual retardation could be found, the achievement in arithmetic seemed more strongly related with problems of personal adjustment than with intelligence or school experience. Overprotection on part of elderly parents seemed to play an important role in some of the difficulties of adjustment.

Some of the overprotected children tried to assert their independence by being overcritical of their classmates and sensitive about themselves; they were usually rejected by their group. They also had a hard time to accept any failure on their own part. Others, who did not rebel against the overprotection, took an attitude of fearful rejection against the social values of their contemporaries. They became the "sissies," the overly anxious and withdrawn group. They had little contact with reality. Neither group is sufficiently detached from their own conflicts to be unpreoccupied for their work. Reading is a favorite flight from reality for them.

Overprotection is but one factor. Rigid demands or high expectations of the achievement of their children create a rigid attitude towards themselves in another group of children showing some retardation, sometimes resulting in defeatism. The family constellations illustrate this assumption:

Six of the thirteen children of the retarded group are the youngest in a group of much older siblings (some of them therefore evidently overprotected). Two children, who are the oldest in their families, formed their demands on themselves in reaction to their mothers' excessive demanding and critical attitude. The remaining five are without siblings. Some

suffer from overprotection and a feeling of too high expectation on the part of their parents, while others suffer from loneliness and the lack of ability to stand competition. The writer's observations are supported by Levy's studies, which show a similar train of thought.

It also seems that the personality of the teacher plays a decisive role in the self-acceptance of these children and in their adjustment to the demands of school.

The insecure children show a definite discrepancy between their scores in reasoning and computation in their achievement tests. They can neither stand the competitive atmosphere that goes with computation nor the emphasis on speed while they are trying to be accurate.

In those very few children whose slow or inadequate reasoning accounts for the retardation, but who are on their grade level in mechanical computation, the retardation, as shown by achievement tests, has increased as the children were promoted into higher grades. This seems to indicate that the number concepts necessary for basic operations were established but that not enough insight was gained for more complicated processes. There better teaching could prevent failure.

It is impossible here to elaborate on the personality structure of the group who finds great satisfaction in their work in arithmetic, though it seems equally challenging to explore. This will have to be reserved for later study.

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The Envelope of a Line Moving Between Two Concentric Circles

By MILDRED HOPKINS
Kankakee High School, Kankakee, Illinois

CURVES formed by a moving line are of great interest to high school pupils. They like the idea that a curve can be formed by drawing its tangents, as well as by joining a number of its points.

During the 1946 meeting of the Mathematics Institute at Duke University, I became interested in the way that string designs can be used to illustrate the envelope of a moving line. Since that time, the geometry students of Kankakee High School have experimented with string designs which connect corresponding points of two concentric circles.

These points were marked off uniformly upon the two circles according to a fixed ratio $(n+k):n$, with n points on the inner circle and $(n+k)$ points on the outer circle. Experiments, with a number of values of n and k in the $(n+k):n$ correspondence of points, indicated that the envelope of the line was determined by the ratio used.

When the line moved around both circles in the same direction, a curve was formed with cusps similar to those of an epicycloid. The number of cusps was the same as the value of k. However, when the line moved around one circle in a clockwise direction and counter-clockwise around the other, the number of special points determined was $2n+k$. These were similar to those of a hypocycloid.

With these observations in mind, the equations of the envelopes can be obtained.

In the first case, the general equation of the moving line l can be written as

$$(1)\qquad \frac{y-ar\sin n\theta}{ar\sin n\theta-br\sin(n+k)\theta}=\frac{x-ar\cos n\theta}{ar\cos n\theta-br\cos(n+k)\theta},$$

where ar is the radius of the larger circle and br the radius of the smaller circle.

In one of its positions, line l connects point P of the larger circle to point P' of the smaller. (See Fig. 1.) The origin is taken at the common center O of the circles OP makes angle $n\theta$ with the x-axis and OP' makes angle $(n+k)\theta$ with the x-axis. That is, two points on line l are $(ar\cos n\theta,\ ar\sin n\theta)$ and $(br\cos(n+k)\theta,\ br\sin(n+k)\theta)$.

By use of the methods of the calculus, the equations of the envelope of line l are found to be

$$(2)\qquad x=\frac{abr[-bn\cos(n+2k)\theta+a(n-k)\cos(n+k)\theta+b(n+2k)\cos n\theta-a(n+k)\cos(n-k)\theta]}{2[(a^2n+b^2n+b^2k)-ab(2n+k)\cos k\theta]},$$

$$y=\frac{abr[-bn\sin(n+2k)\theta+a(n-k)\sin(n+k)\theta+b(n+2k)\sin n\theta-a(n+k)\sin(n-k)\theta]}{2[(a^2n+b^2n+b^2k)-ab(2n+k)\cos k\theta]}.$$

When $k=1$, $n=1$, $a=2b$, and $b=1$, we have the envelope of line l for the 2:1 correspondence of points between the larger and smaller concentric circles. (See Fig. 2.) When the functions are then expressed in terms of θ, the equations of the envelope are

$$(3)\qquad x=\frac{r}{3}\left(\cos 2\theta+2\cos\theta-\frac{1}{1-\cos\theta}\right),$$

$$y=\frac{r}{3}(\sin 2\theta+2\sin\theta),$$

with a cusp at the point $[-r/3,\ 0]$.

By use of the methods used by Cayley in studying caustics of a circle (see

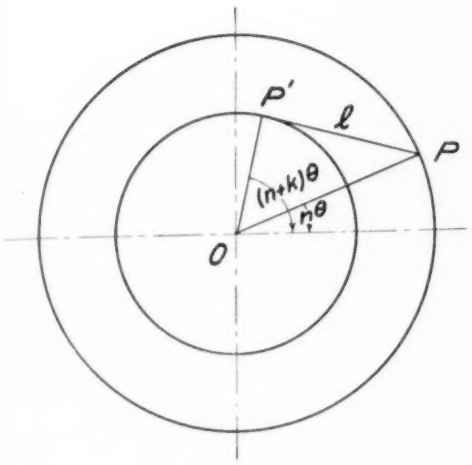


FIG. 1

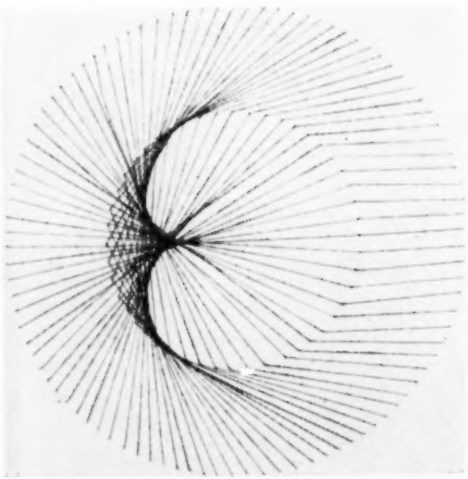


FIG. 2

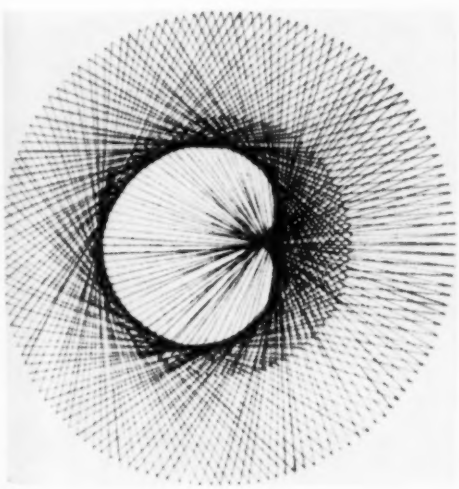


FIG. 3

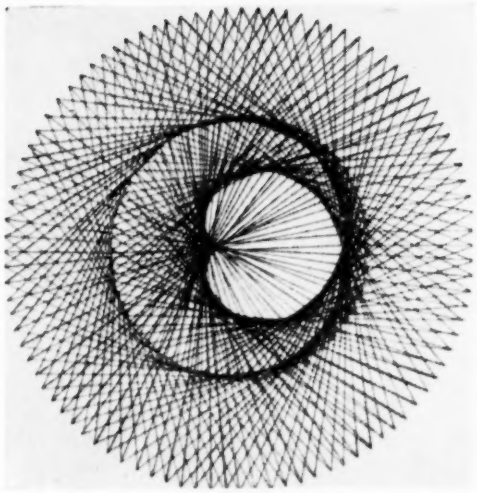


FIG. 4

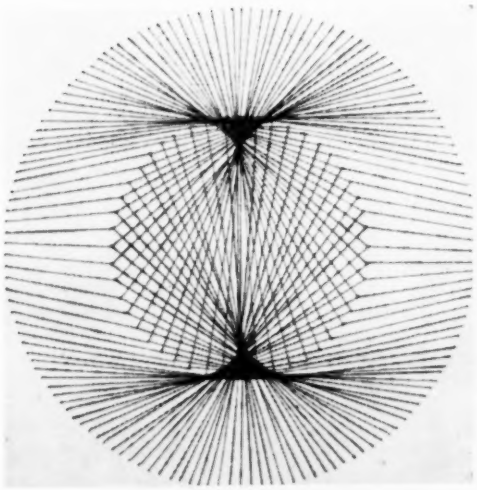


FIG. 5

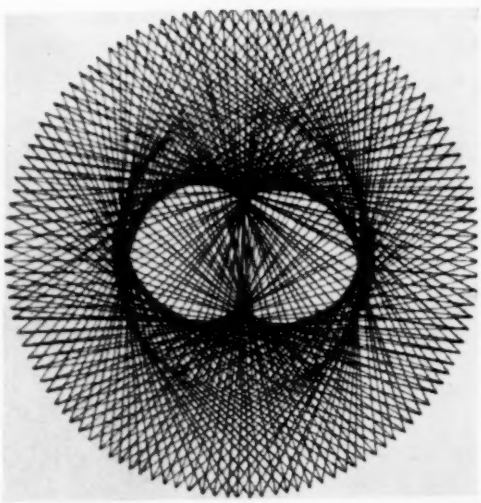


FIG. 6

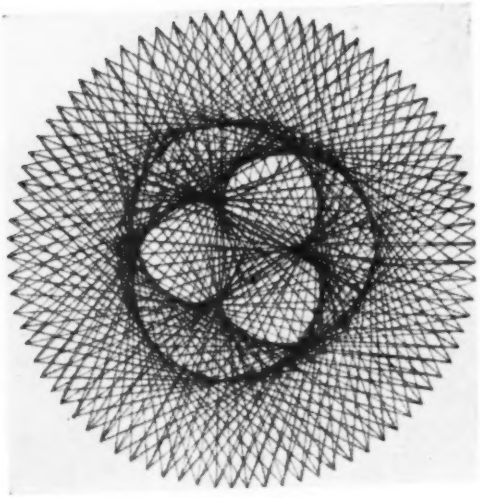


FIG. 7

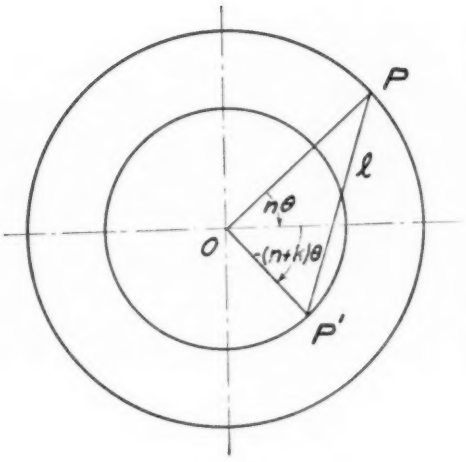


FIG. 10

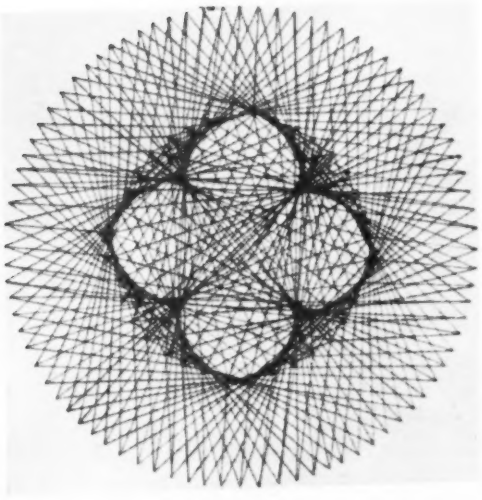


FIG. 8

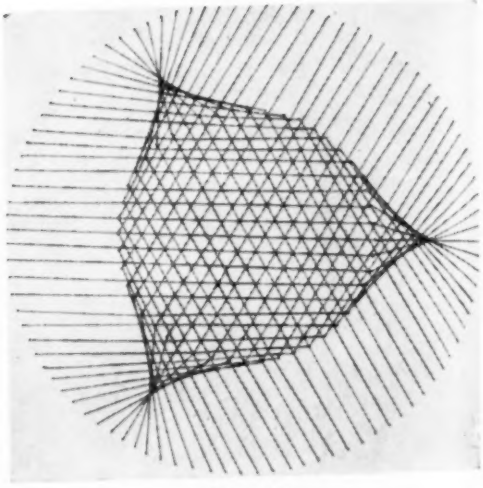
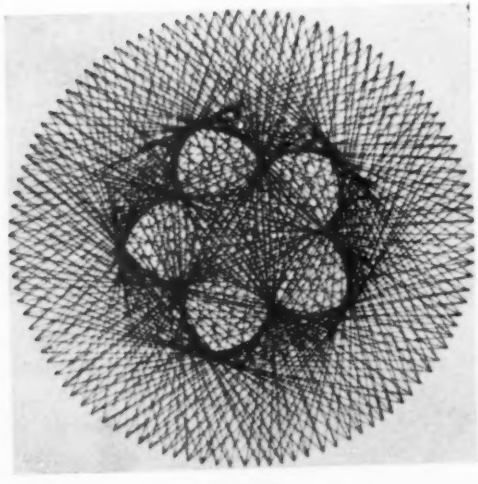


FIG. 11



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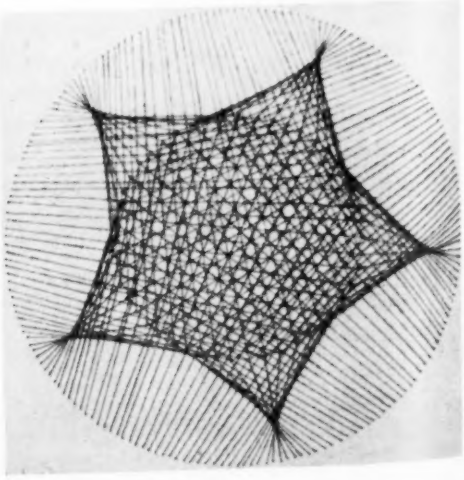


FIG. 12

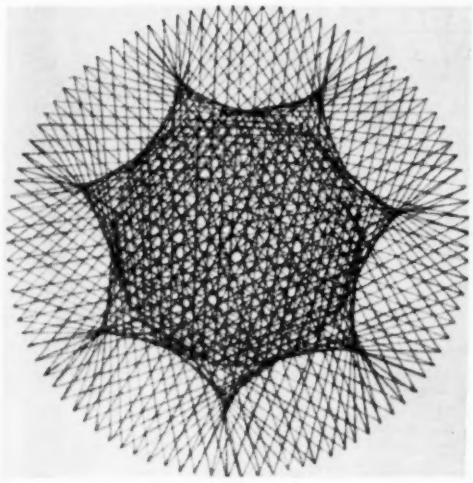


FIG. 13

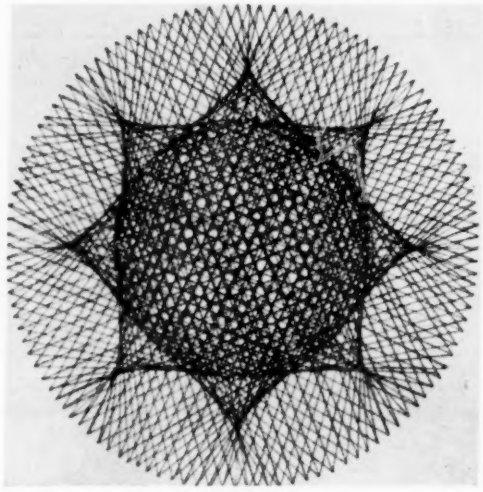


FIG. 14

Cayley's Memoirs), the equation of the envelope of line l in rectangular coordinates is found to be

$$(4) \quad 4r^3(2x+r)^3+27y^2(y^2+x^2-r^2)^2=0.$$

By substitution, it can be shown that equations (3) satisfy equation (4). This curve has one cusp similar to that of a cardioid, an epicycloid of one cusp.

The correspondence 3:2 of points of the two concentric circles is obtained, when $k=1$ and $n=2$. (See Fig. 3.) The equations for this case can be obtained by substituting these values in equations (2). A simple case is obtained by using $a=2b$ and $b=1$.

For the correspondence 4:3, we get similar results when $k=1$, $n=3$, $a=2b$, and $b=1$. (See Fig. 4.)

As n varies and $k=1$, we get a curve with one cusp similar to that of a cardioid. However, the envelope is not a closed curve but continues on in spirals. As n increases, the part which resembles a cardioid gets smaller and the spirals have more revolutions.

For the ratios 3:1 and 5:3 (see Figs. 5 and 6 respectively), we observe that two cusps are formed similar to those of a nephroid and the envelope of l continues on in spirals. For the ratio 7:4 (see Fig. 7), we see three cusps similar to those of an epicycloid of three cusps. When the ratio 7:3 is used (see Fig. 8), four cusps are are obtained. Five cusps are obtained, when the 9:4 ratio is used. (See Fig. 9.)

The equations for all of these special cases can be obtained from equations (2). As indicated above, the spirals have more revolutions as n increases and the number of cusps is the same as the value of k.

In the second general case, line l moves clockwise around one circle and counterclockwise around the other circle. The equation of line l can be written as

$$(5) \quad \frac{y-ar\sin n\theta}{ar\sin n\theta+br\sin(n+k)\theta}=\frac{x-ar\cos n\theta}{ar\cos n\theta-br\cos(n+k)\theta},$$

where ar is the radius of the larger circle and br the radius of the smaller circle. OP makes angle $n\theta$ with the x-axis and OP' makes angle $-(n+k)\theta$ with the x-axis. (See Fig. 10.) That is, two points on l are $(ar\cos n\theta, ar\sin n\theta)$ and $(br\cos(n+k)\theta, -br\sin(n+k)\theta)$.

By use of the methods of the calculus, the equations of the envelope of line l are found to be

$$x=\frac{abr[-bn\cos(3n+2k)\theta+a(n+k)\cos(3n+k)\theta+a(3n+k)\cos(n+k)\theta-b(3n+2k)\cos n\theta]}{2[(a^2n-b^2n-b^2k)+abk\cos(2n+k)\theta]},$$

(6)

$$y=\frac{abr[bn\sin(3n+2k)\theta+a(n+k)\sin(3n+k)\theta-a(3n+k)\sin(n+k)\theta-b(3n+2k)\sin n\theta]}{2[(a^2n-b^2n-b^2k)+abk\cos(2n+k)\theta]}.$$

In illustrations 11, 12, 13, and 14, we see the envelopes of line l for the ratios 2:1, 4:1, 4:3, and 5:3 respectively.

The $2n+k$ special points are similar to the cusps of a hypocycloid. These special points are formed between the circumferences of the two concentric circles, while they were inside the smaller circle in the first case.

It is interesting to observe, that in illustrations 2 and 11, the same ratio 2:1 is used. However, since the line does not have the same kind of motion in 2 as in 11, different curves are obtained. In general, in the first case, the number of cusps is equal to the difference of the terms of the ratio. In the second case, the number of cusps is equal to the sum of the terms of the ratio.

NOTE: the string designs were photographed by John Strasma, a solid geometry student.

Planning a Mathematics Tournament

By EDNA M. FELTGES

Woodrow Wilson Branch, Chicago City Junior College, Chicago, Illinios

"THREE friends ate lunch together. One spent eighty cents for a glass of milk, two hamburgers, and a doughnut. The second bought two glasses of milk, one hamburger, and two doughnuts for seventy cents. If the third bought one glass of milk, one hamburger, and one doughnut, how much did he pay?"

Have you ever seen six or seven hundred men and women, young and not so young, from all over the United States and Canada, madly working such problems for an hour and a half and apparently enjoying themselves? No? Then you were not present in the Gold Room of the Congress Hotel in Chicago on April 13, 1950, when the mathematics department of the Woodrow Wilson Branch of the Chicago City Junior College held its Ninth Annual Mathematics Tournament as a part of the program of the Twenty-Eighth Annual Meeting of the National Council of Teachers of Mathematics.

Why do we hold such contests? They are held chiefly because there is so much discussion about the improvement of methods of instruction for the dull and the average pupil, and so little discussion about the methods of detecting those who have a special interest and ability in mathematics and of encouraging them to continue their studies. This neglect, combined as it is with the loss of potential mathematicians due to World War II, has led to a serious dearth of men and women in basic research, in the teaching profession, and in the applied sciences. We think a mathematics tournament is one way to help renew the supply. If you agree with us and if you have never seen such a contest in operation, this in detail is how it is done.

PREPARATION FOR THE TOURNAMENT

About three months before the tournament, the chairman of the problem committee asks the other members of the department to contribute questions. These are submitted on small cards so that they may easily be sorted for subject matter and order of difficulty. Many of these questions are the brain children of mem-

bers of the department, others have been found in old English and American textbooks, in West Point examinations, or in similar sources. The problem chairman arranges these in eight sets of three problems each and two or three run-off sets of two problems each and presents them to the other members of the department for their criticism as to content, wording, and order of difficulty. There are usually several revisions before a satisfactory complete set is obtained. Then enough copies are mimeographed for all who attend the contest, onlookers as well as contestants; keys are prepared for the graders; and a time schedule is planned.

Two months before the date of the tournament letters are sent to the principals of the public high schools in the neighborhood of the College inviting each of them to enter a team of four students. Three weeks later, entry blanks are mailed to the chairmen of the departments of mathematics with the request that they be returned at least three weeks before the tournament so that the names of the participants may be printed on the programs and may also be sent to the newspapers for publication. On the entry blank is the name of each contestant, the courses which he has pursued in high school, and the names of the members of the mathematics faculty who wish to accompany the team.

To be eligible to compete in the tournament, each student must have had at least one and one-half years of algebra and one year of geometry or he must have had one year of algebra and one year of geometry and be enrolled in third-semester algebra. Of the fifty-two who entered in 1950 only one had the minimum requirement of two and one-half years, seven had three years, twenty-one had three and one-half years, and twenty-three had four years of secondary mathematics.

As soon as the names of the contestants are received, the chairman of the department prepares a statement for the press attaching to it several problems used in the previous tournament, their solutions, and the names of the contestants from each of the thirteen teams. News reports and pictures of contest winners appear in all the large Chicago daily papers and in the neighborhood papers both before and after the tournament.

Conduct of the Tournament

Contestants are seated by teams in the front of the room and their sponsors and other guests in the back. The faculty members and students who grade the papers sit in chairs in the front of the room facing the audience. They are supplied with grade cards for each team and on these a cumulative record is kept so that at the end of the tournament it will be necessary only to compare cards to pick the winners.

After a short greeting to the competitors, the rules are explained by the tournament conductor and the procedure illustrated by means of a sample problem given to each contestant. When this problem is worked the answer is checked, but no grade is recorded. Then mimeographed sheets containing three problems each are passed down the aisles and placed face down. At a given signal the papers are turned over and the contestants are off on the first lap of the race.

After a lapse of from two to twelve minutes, depending on the difficulty of the questions, a warning signal is given. One minute later, the signal "Stop" is given, pencils are put down, and the graders pass along the aisles marking the answers right or wrong on the papers and putting the scores on the grade cards. It takes only two or three minutes to check eight papers, collect them, and distribute the next set of questions.

During the first two years the plan was to have each contestant drop out and move to a rear seat as soon as he had missed five questions, but that procedure embarrassed the less fortunate competitors and their teachers, created confusion, and consumed considerable time. Now each contestant competes in the first eight relays. If, at that time, there are no ties for either the four highest teams

or the six highest individuals, the tournament is finished and the guests and students repair to the lounge for tea and cakes and for the presentation of the awards. However, it seldom happens that ties do not have to be resolved. When teams are tied, additional relays of two problems each are given to everyone until one team forges ahead. Only the tied teams are graded and no one but the judges knows which teams are still competing. If individuals are tied at the end of the eighth round, they are scored in the team run-offs. If ties still exist, all the problems are weighted; each question has the same weight as the number of the round. The time for the entire tournament is usually less than two hours.

The individual awards, six in number, are Pythagorean medals inscribed with the winning place, the date, and the college name. During the war when metal was scarce the awards were books on mathematical subjects but the winners seem to prefer medals which they can wear and display to their classmates. The team awards are certificates of merit which they can post on their bulletin boards. In 1950, however, since the contest was held before several hundred delegates of the National Council of Teachers of Mathematics the awards were more numerous and more elaborate. A full-year tuition scholarship to the University of Chicago, subject to renewal under the usual conditions, was presented to the individual first-place winner. A second scholarship for one year was awarded by the Technological Institute of Northwestern University to one of the high-ranking contestants who met the entrance requirements for engineering work. Also among the awards were many valuable articles donated by companies which are interested in furthering the study of mathematics. These included drawing sets, portfolios of photographs of famous mathematicians and physicists, mathematical postcards, slide rules, and books. Each contestant received a slide rule; the books were presented to the two highest on each team; and at the end of the eighth round candy bars were distributed to supply energy for the run-offs.

Conclusion

In addition to encouraging potential Lisa Meitners and Harold Ureys of tomorrow to continue their studies in mathematics and science, there are many valuable outcomes of the tournament, a few of which I shall list here.

1. The tournament motivates the teaching of the high school instructor since in most of the competing schools preliminary tournaments are held before the team is chosen. This opportunity for mental gymnastics usually inspires many more students than the four who enter the contest.
2. The public is made aware, at least once a year, of the importance of mathematics both in our everyday living and in our survival as a free and prosperous nation. Groups in factories and offices work the problems which they see in the newspapers and telephone us to settle their disputes when their answers differ. One bed-ridden invalid amused himself for days working a set of problems and then sent for several more sets.
3. Mutual benefits result from the close cooperation of the high school and the junior college instructors.
4. The news releases inform the public of the existence of a tuition free city college which any high school graduate may attend at small expense.
5. A few contest winners already show promise of attaining high positions in mathematics or science. Many of them are still doing graduate work in some university.

These contests mean a lot of hard work but the results seem to justify all the time and effort spent on them. It is hoped that there will be more of them and that they will lead to city-wide or state-wide contests.

Notice to Members

To insure prompt delivery of your Mathematics Teacher, please send any change of address to The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C. Be sure to give the *old* address as well as the *new*.

A Japanese Version of the History of Mathematics

By Edward Kasner *and* Irene Harrison
Columbia University and New York University

During the last year we have had extensive correspondence in English with a young Japanese engineer, J. Okita, regarding translation of "Mathematics and the Imagination" into Japanese.

Perhaps an excerpt from his most recent letter will be of interest to teachers of mathematics. This particular excerpt really gives an aperçu of the history of mathematics from the viewpoint of a self-educated mathematician of some originality and poetic insight. We quote it exactly as written.

* * *

Statical, Mechanical and Dynamical Mathematics

To be brief, Mathematical History is divided into three ages, the ancient age, the modern age and the extremely recent age, by my self-satisfied classification.

Those are shown in my poetical expression, as following. The mathematics in the ancient shutted itself up in statical moon light. The mathematics in the modern was a mechanical march, both in its manner and its mood. The mathematics in the recent years is dynamical. If I approach it, I am thrown out right away.

Here I mean that the ancient is the ages of Greek, India, Arab; the modern is the term from 17th century to 18th century inclusive; and the recent is after 19th century, particularly 20th century.

Now in Greece, the idea and the conception of philosophers, for instance Plato's idea, was static and quiet. Euclid, Pythagoras and many other scholars treated "Statical, unmoved figures"—though only Archimedes and others were different somewhat. Algebra in India and Egypt were statical comparing with the modern algebra, I think. So I would say that the ancient is statical.

The modern mathematics (it is called classic mathematics) mainly began with Descart's analytic geometory in the early days of 17th century and calculous discovered by Newton and Leibniz followed to Descart.

Now, the method of algerbra is mechanical. Thus, given an equation of one degree. To solve it, we arrange and transfer the unknown quantity to the left, known quantity to the right, next we divide with the coefficient of unknown quantity. Although we somewhat need a skill, we can solve mechanically when we became familiar with it. We need not to deliberate and ponder these problems with mental activity such like geometory of Euclid.

It is an important thing that Descart and Newton gave us the procedure which the former treated mechanically geometoric idea and the latter mechanically dealed with kinetics and motion object. Since Renaisance this mechanical mathematics furnished the mechanical application for all the science and industrious world. So I would say that the mathematics in the modern is mechanical.

In the 19th century, mathematics was altered again. In this century the analytical mathematics made a great progress, but it was chiefly in mechanicalization, and technique neglected the logic itself.

I believe, the 19th century can be said a period which accomplished the analytical mathematics and at the same time a period which the mathematicians had to introspect mathematical logic.

The mathematics of the 20th century is dynamic. Of course, I have not any qualification to discuss about it. But I feel it dynamically, because it seemed to deal with motive object, or not to allow mechanical procedure, or to need dynamical mental activity, etc. Furthermore, the mathematics in the 20th century produced a dynamic bomb (atomic bomb). Alas! If

I approach to learn the mathematics in recent years, I am thrown right away by its dynamical power. So I would say that mathematics in recent years is dynamical.

—end—

* * *

The amazing thing about this is that Mr. Okita claims nothing for the Japanese. Yet years ago many Japanese historians made rather extravagant claims about their early mathematical discoveries including calculus and determinants. Apparently this young gentleman gives all the credit to Greeks, Hindus, Arabs and modern Europeans. This is all the more surprising as Mr. Okita lived quite near Hiroshima at the time of the atomic bomb. As a matter of fact he heard the explosion. In spite of this he writes in a delightfully objective and philosophical way.

Teaching the Slide Rule via Television

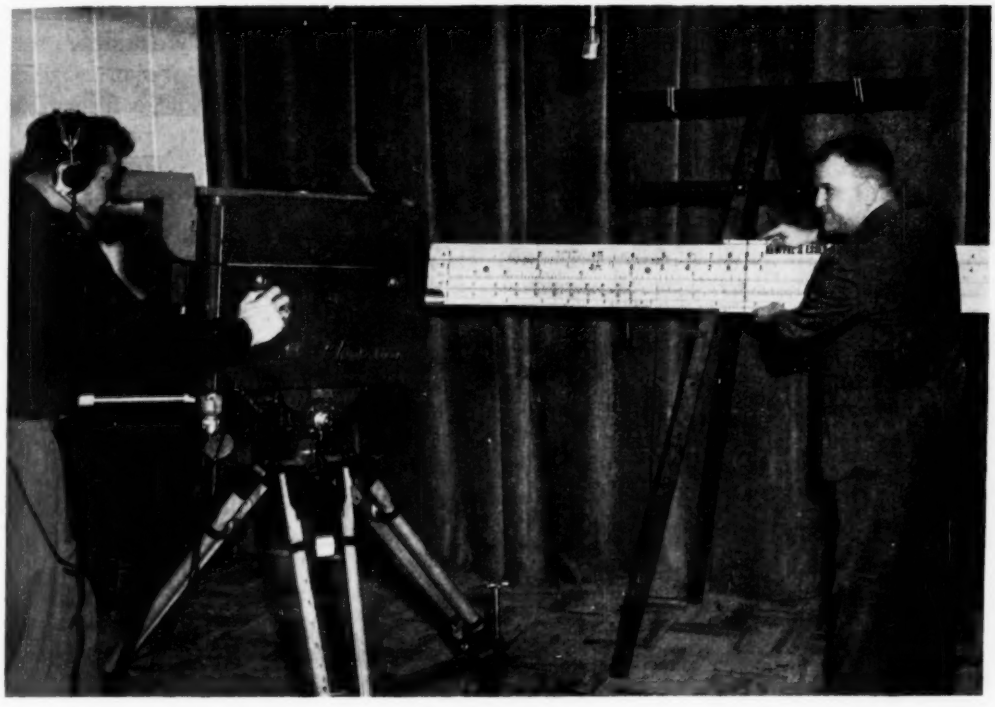
By GEORGE R. ANDERSON
State Teachers College, Millersville, Pa.

SINCE October, 1949 Millersville State Teachers College has been presenting a series of weekly programs, fifteen minutes in length, from station WGAL-TV, Lancaster, Pennsylvania. These programs are, in words frequently used on the program, 'designed to add to your general information and to suggest certain things which you may make or do." In spite of admitted limited audience appeal, the station's program director, Mr. Paul Rodenhauser, generously agreed to a series of six programs dealing with the slide rule. The purpose of the series was to acquire techniques of presenting such instruction and to observe as much as could be learned about "student" reaction to this method of instruction. When the series was first conceived, it was planned to organize typical listener groups in various schools and industries along with a small studio class. Because the production of these programs is entirely an extra-curricular activity for the writer, it was found impossible to devote the additional time required for such groups. A test administered to each of these groups might have given valuable evidence as to the effectiveness of the presentation. It is to be hoped that others repeating the experiment will have the facilities and the time to make such observations.

The first program began with a brief sketch of number devices illustrated with Napier's Computing Rods and an abacus. Then mention was made of the use of fingers in computation particularly with reference to multiplying numbers from six to nine inclusive.[1] It was pointed out that our decimal system was a physiological "accident" and that there are systems of notation other than our decimal system. Here the slide rule was introduced with emphasis not only on its computional value but that it is also a convenient method of presenting data. To illustrate this a photographic and a musical slide rule were introduced along with a navigational computer (Dalton Mark VII). The problems considered here were as follows:

1. In making a stop bath for photographic prints it is necessary to use 3 parts of acid to 8 parts of water. If I want to

[1] Underwood, R. S. and Sparks, Fred W. *Living Mathematics*, McGraw-Hill Book Company, 1949, p. 5 and 6.



Mr. Roy Swinnimer of WGAL-TV, Lancaster, preparing to telecast Professor G. R. Anderson's slide rule demonstration.

make 16 oz. of stop bath, how much acid should I use?

2. Change the following scores to percentages if 267 is a perfect score: 166, 208, 199, 240, 251, 144.
3. I drove 1,389 miles on 101.4 gallons of gasoline. What was the number of miles per gallon?
4. Our basketball team won 47 out of 63 games during the past several seasons. What percentage of wins did this represent?

No effort was made to teach on this first program. The sole objective on the initial program was to create an audience by arousing their interest in the possibilities of the slide rule. It was gratifying to learn that youngsters as well as older persons who, "always wanted to learn how to use the slide rule," were equally intrigued.

Programs two and three were concerned largely with learning to read the rule. Since the studio was small and there were only two cameras available it was decided to use the film, THE SLIDE RULE—C AND D SCALES, produced by the U. S. Office of Education. This film has excellent close ups and good animated drawings both of which are vital to a good television presentation. Ten minutes of this program were film presentation followed by a demonstration of these two problems on multiplication.

1. What will 26 quarts of milk cost at 20½¢ per quart?

2. How much pay will I get for 21 hours at $1.63 per hour?

Eight minutes of film introduced program three. Typical problems in division were used to complete the time.

Program four was a practice program involving multiplication and division using only the C and D scales.

Since program five fell on St. Valentine's Day it was introduced by this equation accompanied by its graph.

$$y = \sqrt[3]{x^2} \pm \sqrt{100 - x^2}.$$

The graph, as you may know, is heart shaped.[2]

On program five the CI and the A and B scales were also presented. To illustrate a problem in square root it was explained that the normal focal length of a lens is equal to the diagonal of the negative which it is intended to cover. A negative four inches by five inches was scored along the diagonal and the length of this diagonal computed.

In the sixth and last program proportion was the main theme. Problems sent in by viewers were discussed. Typical of these were:

1. A picture $9\frac{1}{2} \times 7\frac{1}{4}$ was to have a length of 5 inches in a yearbook. What will be its width?
2. Ribbon costs 42¢ per yard. What will 55 in. cost?

Many of the problems of radio instruction still plague that of television. Perhaps the greatest of these is timing. Since programs were given at 7:30 in the evening it was impossible to have regular school classes participate except on a voluntary basis in their homes. Teachers cooperated by loaning rules for home practice. Some may feel that since the film was used it would be less expensive and just as profitable to show the film directly in the classroom. In using the film on television the comments that one may make in the ordinary classroom can be interspersed on the television program and the writer feels that a good learning atmosphere can be made to prevail. Such instruction will not replace any classroom teacher but, like other audio-visual aids, will supplement and enhance the present classroom program. The technique should be explored in many fields if for no other reason than as a means of instruction in a national emergency when time is a vital factor in training many people for a specific task. Television can be helpful when detailed instructions are to be made to a large group. Schools of medicine find it unusually good when color television is employed. The writer has observed a program demonstrating insect repellents in which the fly used to test the effectiveness of various concentrations appeared on the television screen to be five inches long. The results were more clearly observed than would have been possible if one were present in the room with the demonstrators. The writer feels the impact of television on the educational program will be vastly greater than that of radio.

As a final observation it was gratifying to note the fine public relations afforded by television. Many persons who never before saw a slide rule at least learned enough about it to remove the mystery element. We who love mathematics sometimes forget that the mathematics program must be continuously sold to the general public.

[2] Boyer, Lee E. *Mathematics, A Historical Development*, Henry Holt and Co., N. Y. 1945, p. 223.

POP = X

Let *POP* stand for our Postcard Opinion Poll and let *X* stand for your answers on a Penny Postcard.

We would like our readers to send us their answers to the following questions:

A. Which article did you like best in this October issue? (Simply list the letter A followed by the number of the first page of this article. Second and third choices are also welcomed.)

B. Which department did you like best in this October issue? (List the letter B followed by the number of the first page of this department. List second and third choices also, if you wish.)

C. Which book(s) on the Books Received list would you like to have reviewed in a forthcoming issue? List the author(s) only.

D. Please list the date on which the October issue of THE MATHEMATICS TEACHER reached you.

Mail your card to the Editor within seven days of receipt of your TEACHER.

APPLICATIONS

Edited by SHELDON S. MYERS
University School, Ohio State University, Columbus, Ohio

THIS department is an effort to help meet the need for sources of enrichment and motivation at all levels of elementary and secondary mathematics. It is the opinion of the writer that applications may not only motivate but also supply some of the learning experiences whereby mathematical insights and understandings are acquired by the students. Applications need not be geared only to the gifted student. This department will attempt to describe applications which are suitable for wide ranges of ability at different grade levels.

Contributions from teachers and students are earnestly sought, and due recognition will be given for each application used. Please send them to the writer of this department with appropriate references in case the application is from a book or magazine.

In case readers wish to inquire further about particular applications, we are adopting the following system of classification and numbering: Ar. 1 Gr. 6–8 Arithmetic, first application, grades 6 to 8; Al. 3 Gr. 9–10 Algebra, third application, grades 9 to 12; C. Al. College Algebra; G. Geometry; S. G. Solid Geometry; T. Trigonometry; P. Proof; A. G. Analytic Geometry.

It is to be hoped that this department will be of use in suggesting how settings with local color and interest can be capitalized on for the formulation of problems close to student interests. Here is an example of what we mean:

Ar. 1 Gr. 7–8 MATHEMATICS OF WAVES
(Sample study guide)

One evening at the seashore we were counting the waves rolling in and discovered that every eighth wave was larger than the rest. Can you answer the following questions that might arise at the beach?

1. After a large wave had passed, we began counting and stopped when we reached 56. Assuming that our discovery still holds true, would the 56th wave be a large one? Why? How about the 33rd wave?

2. One evening at sundown our wristwatch second hand told us that a wave broke on the beach every 3 seconds. We counted 50 *large* waves while we were watching the sunset. About how many minutes were we there?

3. One evening when the waves were breaking every 3 seconds, we estimated that the crests of the waves were about 9 feet apart. About how many feet per second were the waves traveling?

4. If the waves were traveling at the above speed, how many seconds would it take a wave to travel the length of a 90-foot jetty at right angles to the waves?

5. Does water travel with a wave? Try to find out with a match stick. What is it that moves along with a wave?

Now that beginning algebra classes are starting again in the fall, we will be confronted once more with the perennial question "What is algebra?" We cannot describe the entire course in reply. Nor can we employ the noncommittal, "Wait and see" reply. Simple formulas seem to be the easiest way to get at some of the fundamental meanings of algebra in the first few days. Simple formulas bring out the meaning of algebra as a shorthand language describing related variables, bring in the notion of a general number, illustrate elementary algebraic operations, and show immediately one of the uses to which the subject is put. Here are some useful formulas, some of which come from the Seventeenth Yearbook of the National Council of Teachers of Mathematics:

Al. 1 Gr. 8–9. SIMPLE FORMULAS

1.1 $$h = 8 + \frac{18 - a}{2}$$

Where

a = age in years up to 18

h = hours of sleep you should get

Problems:

How many hours of sleep should you have?

If you have a little brother or sister, how much sleep should they have?

1.2 $$d = \frac{n}{5}$$

Where

n = number of seconds between the lightning flash and the sound of thunder

d = number of miles the lightning is away

Problems:

If you were watching fireworks at an amusement park and .2 seconds elapsed before you heard the sound, how many feet away were you?

Will this formula work with heavy artillery fire?

1.3 $$S = 16T^2$$

Where

T = time in seconds

S = distance a body has fallen without air resistance

This formula can be verified by a class using stop watch and tape measure by dropping weights through measured distances alongside a school building. First, second, and third floor distances are sufficient.

1.4 $$M = 6545D$$

Where

D = diameter of tire in feet

M = expected mileage from the tire of average automobile

Valuable discussions can be stimulated with this formula in regard to how it was derived and whether it is up to date. Students can write to the tire companies for a more recent formula based on recent tires.

A problem which occurs widely in industries and especially in laboratories is the following:

Ar. 2 Gr. 9–12 ALIQUOTING (Meaning illustrated by the problem)

Ten grams of a substance are dissolved in 100 ml. (milliliter) of reagent. Of this solution, 75 ml. are diluted to 300 ml. Out of the 300 ml., 100 ml. are taken and diluted to 200 ml. How many grams of the original substance are left in 10 ml. of the final solution? (Answer next month)

Here is a class project that could arouse quite a bit of interest in your trigonometry class. To get the necessary data, your students could contact a trigonometry class several hundred miles due north or south of them. This project can be found in *Mathematics for the Million* by Launcelot Hogben and other sources. It was first performed by Eratosthenes, a Greek geographer, in Egypt.

T. 1 Gr. 11–12 MEASURING THE EARTH'S CIRCUMFERENCE

A large, measured pole is used. The length of the shadow of this vertical pole is measured at two points located on a meridian and separated by several hundred miles. The shadows are measured as near to the time when the sun crosses the meridian (noon) as possible. The following data were collected for Kalamazoo and Cincinnati:

Estimated north-south distance Kalamazoo-Cincinnati 240 miles

Length of pole = 82.62 inches

Length of Kalamazoo shadow = 53.99 inches

Length of Cincinnati shadow = 49.80 inches

The solution involves finding the angle at the center of the earth which subtends the two measuring points. The angles between pole and sunrays were computed by the tangent function. The complete solution with illustrative drawing will be given next month.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

THE department *Miscellanea—Mathematical, Historical, Pedagogical* reappears under a revised heading for a year's trial run as a clearing house for notes that are substantially mathematical in nature. Send along your trick problems, neat devices, pet mathematical peeves for all levels from kindergarten through junior college.

Our first item is a carry-over from last year's column. On page 35 of THE MATHEMATICS TEACHER for January 1950 (vol. XLIII) Norman Anning referred passingly to the problem of the "hippopotamus' child" which had appeared in the *Mathematical Gazette*. We are pleased to have the exact reference supplied by T. A. A. Broadbent, Editor of the *Gazette*.

1. *The Rhinoceros Problem*

Just in case anyone is tempted to search for the statement of the "rhinoceros" problem, it is given in the *Mathematical Gazette*, vol. XV (1930–1931) p. 20. Some solutions and comments are given later in the same volume, p. 218.

T. A. A. Broadbent,
Royal Naval College,
Greenwich, London

With Mr. Broadbent's permission we reproduce the original problem which appeared as "Gleaning 718."

"718. In a number of years equal to the number of times a rhinoceros's mother is as old as the rhinoceros, the rhinoceros's father will be as many times as old as the rhinoceros is years old now. The rhinoceros's mother is twice as old as the rhinoceros will be when the rhinoceros's father is twice as old as the rhinoceros will be when the rhinoceros's mother is less by the difference in ages between the mother and father than three times as old as the rhinoceros will be when the rhinoceros's father is one year less than twelve times as old as the rhinoceros is when the rhinoceros's mother is eight times the age of the rhinoceros.

When the rhinoceros is as old as the rhinoceros's mother will be when the difference in ages between the rhinoceros's father and the rhinoceros is less than the age of the rhinoceros's mother by twice the difference in ages between the rhinoceros's father and the rhinoceros's mother, the rhinoceros's mother will be five times as old as the rhinoceros will be when the rhinoceros's father is one year more than ten times as old as the rhinoceros is when the rhinoceros is less by four years than one-seventh of the combined ages of his father and mother.

FIND THEIR RESPECTIVE AGES

(For the purposes of this problem, the rhinoceros may be considered to be immortal.)

[Per A. S. E. Ackermann]"

2. *Perbisectors*

We are accustomed to use simple terms for some of the lines associated with a triangle, such as altitudes and medians. The bisectors of the angles are sometimes called angle bisectors. Why not use a short term in place of the long phrase, "perpendicular bisectors of the sides"? For several years, I have been using the term *perbisectors* to mean, "the perpendicular bisectors of the sides." My students like to use it. The advantages are obvious. No objections have been made, except that it is new and original. I hope that other teachers will consider its use, and, if they like it, that they will make general use of it.

Merton T. Goodrich,
Keene Teachers College.
Keene, New Hampshire

3. *A Note on the Pythagorean Theorem*[1]

According to Plato and Plutarch it appears as certain that the celebrated theorem concerning the square on the hypotenuse of a right triangle had been preceded by the particular case of the triangle whose sides are respectively equal to 3, 4, 5 units of length, which is right angled, since $3^2+4^2=5^2$. We do not possess a proof for this special case which dates from ancient times. Nevertheless, such a proof may have been given in that epoch, or since. In any case, the following, which utilizes only elementary geometrical properties which are independent of the Pythagorean theorem, may interest the reader.

THEOREM. *A triangle whose sides are proportional to* 3, 4, 5 *is right angled.*

In figure 1 let I be the incenter of triangle ABC whose sides are $CA=3a$, $AB=4a$, $BC=5a$, and E and H the points of contact of the incircle with the sides CA and AB respectively, D the midpoint of arc AB of the circumcircle of the triangle, M the midpoint of side AB, and I' the excenter of the triangle opposite angle C. If E' is the foot of the perpendicular dropped from I' on CA, then it is known that CE' is equal in length to the semiperimeter, s, of the triangle. Thus $CE'=6a$, and A is the midpoint of CE'. Let the angle bisector CII' cut AB in F. If we can show that F is the midpoint of CI', the line AF, joining the midpoint of CE' to that of CI', would be parallel to $E'I'$, and hence perpendicular to CE', and our theorem would be established.

Now M is the projection of D on AB, and it is known that $CE=s-AB=6a-4a=2a=BM$. Further, angles ECI and MBD are equal, both being measured by one half the arc AD. It follows that triangles BMD and CEI are congruent right triangles. Therefore $DB=IC$. But we know[2] that $DI'=DB=DI$, whence $DI'=IC$. From the congruence of the triangles it also follows that $IE=DM$. Therefore $IH=DM$, and F is the midpoint of DI. It now follows that F is the midpoint of CI', and the theorem is established.

Victor Thébault,
Tennie, Sarthe, France

[1] Translated from the French by Howard Eves, Oregon State College.

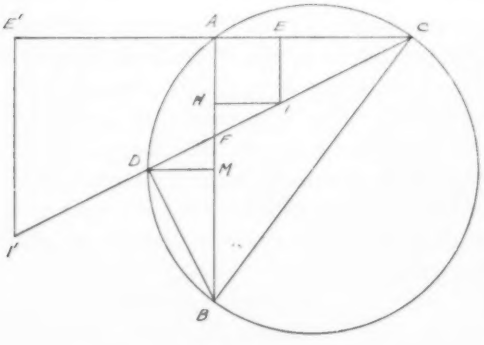


FIG. 1

4. *Trisecting Any Angle*[3]

In figure 2, AOB is any angle. Line OC bisects it. With the vertex of the angle, point O, used as a center, construct many concentric circles. In the smallest circle construct an equilateral triangle $I'EF$ with the side EF perpendicular to and bisected by line OC. In each concentric circle, construct chords E_iF_i equal and parallel to EF and symmetric about OC. On each circle locate points G_i and H_i such that $E_iG_i=E_iF_i=EF=F_iH_i$.

Draw the curves GD and HD through the points G_i and H_i. These curves cross the sides of angle AOB at the points marked x and y. Draw the circular arc xy with O as center. EF used as a chord in this circle will divide arc xy into three

[2] Since $I'B$ would be perpendicular to BI and $I'A$ to AI, $AI'BI$ would then be a cyclic quadrilateral with the center of the circumscribed circle at D.—Editor

[3] Editor's note: We publish a trisection with great misgivings. We shall not be easily induced to repeat the act or to even debate the question. However, this original solution by a high school student deserves some recognition, and presents an elementary approach which some teachers and pupils might use. The last paragraph, which notes the flaw in the solution, is added by the editor but based upon comments in Mr. Todd's accompanying letter Mr. C. E. Jenks was Mr. Todd's teacher.

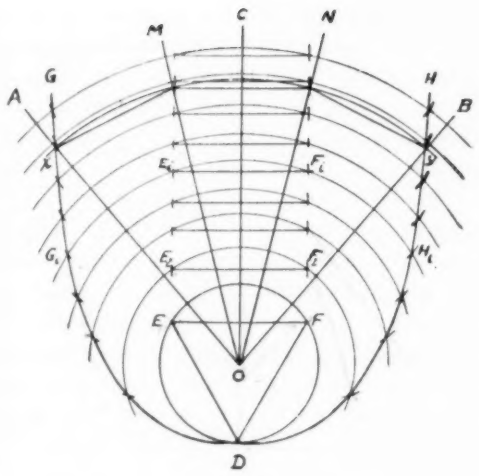


FIG. 2

equal parts. Lines OM and ON through these division points will trisect angle AOB.

The curve which makes this trisection possible can only be constructed approximately with a ruler and unmarked straightedge. It may be determined more accurately by constructing many concentric circles closer together.

Werner S. Todd, Jr., Student,
Wakeman High School,
Wakeman, Ohio

5. *A Geometric Solution of the Quadratic Equation*

John Wallis' geometrical construction of the roots of a quadratic equation is often cited indirectly in histories of the geometric representation of complex numbers. In this context, its possible usefulness in the teaching of mathematics is often ignored by both writers and readers. Such constructions, by pointing out relations between algebra and geometry, help maintain old skills and teach mathematical insight and appreciation. The historical setting of Wallis' construction not only adds to its interest but also gives a concrete illustration of the struggle it took to expand our number system. Students will appreciate seeing the difficulties of the old masters and be cheered by them.

Wallis' directions for the solution of "$aa \mp ba + \alpha = 0$", which from now on will be written as $x^2 \pm bx + c = 0$, were as follows:[4] Take $AD = b$ and construct $PC = \sqrt{c}$ perpendicular to AD at its midpoint. Now take $PB = (\frac{1}{2})b$ and construct a right triangle with it and PC. Use whichever is the longer as the hypotenuse. If $\sqrt{c} < (\frac{1}{2})b$ (i.e., if $PC < PB$ or $b^2 - 4c > 0$), the resulting diagram is shown in figure 3. The roots are represented by AB and BD where both are to be taken as positive or negative depending upon whether the sign preceding b was negative or positive. (Wallis regarded literal numbers as always positive).

If $\sqrt{c} > (\frac{1}{2})b$ the diagram of figure 4 results. The roots in this case are imaginary. It is this latter situation which led Wallis to comment that the points B and B' and the lengths AB, BD, or AB', $B'D$ could be associated with the complex roots. Wallis compared the difficulty of conceiving complex numbers with that of negative numbers. The latter, he said, are impossible "Since that it is not possible that any Magnitude can be less than Nothing, or any Number Fewer than 0. Yet is not that Supposition (or Negative Quantities) either Unuseful or Absurd;

[4] The diagrams and quotations are from John Wallis, *De Algebra Tractatus* as it appears in his *Opera Mathematica*, (Oxoniae, 1693) II, p. 286–292 checked against the English edition of 1685. A portion of the latter may be found in D. E. Smith, *A Source Book in Mathematics*, (New York: McGraw-Hill Book Co., 1929) pp. 46–54.

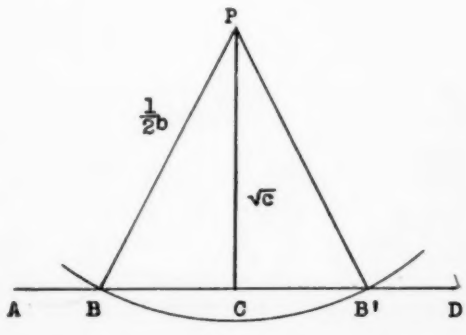


FIG. 3

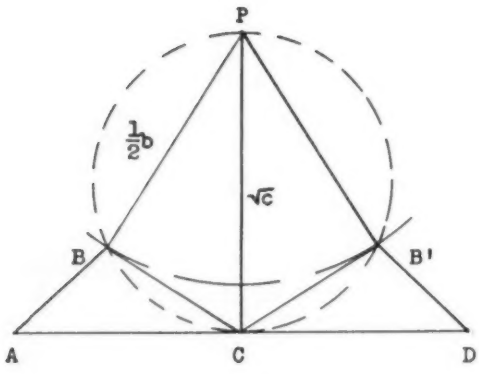


FIG. 4

when rightly understood." Wallis then gives geometric and physical interpretations of negative quantities in preparation for his statement that just as negative quantities may be regarded as laid off on a line "backward," so imaginary roots may be represented not forward or backward along the line as would be real roots, but above the line as in figure 4.

This comparison of the "unreality" of negatives and imaginaries is as useful pedagogically today as it was then. It helps to point out the abstract nature of number. Geometric and physical interpretations (Wallis talked of negative areas—land covered by water) play a role now as then in gaining some popular acceptance of these numbers.

Application and geometric representation alone will not provide a rigorous or complete mathematical theory. Wallis' scheme *could* be extended to provide a representation of every number of the form $a+bi$, but Wallis did not do it. Wallis neither touched upon the inverse problem of associating a number with each point nor did he utilize an imaginary axis even implicitly. Hence it seems that both the bare statements that this was "the first record of an effort to represent the imaginary number graphically by the method now used,"[5] and that "In effect, Wallis represented the complex number $x+iy$ by the point (x, y) in the plane of cartesian coordinates; what he missed was the use of the y axis as the axis of imaginaries"[6] give a slightly exaggerated impression of Wallis' accomplishment.

Wallis' solution of $x^2 \pm bx - c = 0$ is of interest for its utilization of the geometric theorem that the tangent to a circle from an external point is the mean proportional between the segments of any secant through the point. Merely take (figure 5) $CP = (\frac{1}{2})b$, construct $PB = \sqrt{c}$ and perpendicular to CP, draw circle PAD with C as center and draw BCD. AB and BD

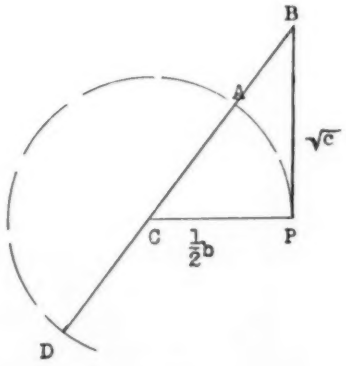


FIG. 5

are the roots. They are opposite in direction and sign, the sign of AB being $+$ or $-$ as is the sign preceding b in the equation. As Wallis notes, algebraically, there can never be an imaginary root in this case and geometrically the construction is always possible.

This discussion suggests the question, "How many ways are there to solve a quadratic equation?" Perhaps some of our readers can send us some suggestions. It would make a good mathematics club topic, too.

[5] D. E. Smith, *History of Mathematics*, (Boston: Ginn and Co., 1923) II. p. 408.

[6] E. T. Bell, *The Development of Mathematics*, (New York: McGraw-Hill Book Co., Inc., 1940) p. 161.

AIDS TO TEACHING

Edited by

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BOOKLETS

B. 33—How to Invest

Merrill Lynch, Pierce, Fenner and Beane, 70 Pine Street, New York 5, N. Y.

Booklet; 32 pages; $8\frac{1}{2}'' \times 11''$; 1948; free

Description: This booklet is written to promote prudent investment of savings in stocks and bonds. It begins by stressing the need for a planned financial program. After describing different types of investments, it emphasizes the need for a diversified savings program. It describes the various types of business organizations, the differences between stocks and bonds, and compares common and preferred stocks. The different types of investments are compared on the basis of growth, income, and safety. The business trend of a given company is explained by means of graphs and by the computation of the price-earning ratio of its stock. These measures of business conditions are used to advise the investor as to what stocks to buy and sell—and when. The booklet recommends that the economic position, financial condition, management, and price should be basic considerations in the selection of the company in which to invest. It discusses where the reader can get information about corporations and how to open a securities' account. It includes a bibliography and glossary of terms.

Appraisal: This booklet was written for the adult to tell him how to invest and to build his confidence in the buying of securities for investment purposes. Its language and content are not for immature minds. However, the teacher of senior general mathematics or business arithmetic will find in it valuable material. There is little advertising of a specific company but the tenor of the entire booklet is to promote the sales of securities. However, it does emphasize the need for careful investment according to the individual's needs in reliable securities.

B. 34—Mathematics at Work

W. W. Rankin, Duke University, Durham, North Carolina.

Booklet; 127 pages; $8\frac{1}{2}'' \times 11''$; $2.50

Description: This publication is a summary of the highlights of the Ninth Annual Mathematics Institute held at Duke University, August, 1949. It is presented in six major parts, namely: lectures, applications of mathematics, pure mathematics, teaching of mathematics, study groups, and special interests. The lectures present such topics as the mathematics laboratory, famous problems and ideas and the spirit of mathematics. The applications of mathematics are made in such diverse fields as automotive engineering, electronic measurement devices, quality control and color mixture. The section on pure mathematics discusses non-euclidean geometry and applications of number theory. The development of meaning and understanding of basic concepts and principles receives the major emphasis in the chapters on the teaching of mathematics. The material presented by the study groups also emphasized laboratory materials and techniques.

Evaluation: Every teacher of mathematics should attend a mathematics institute at Duke University but if it is impossible to attend the Institute, the bulletins giving the highlights of these

institutes are a good substitute. This pamphlet is full of ideas which every mathematics teacher needs for making his instruction more meaningful. It describes the use of a great many devices and illustrations that should make mathematics palatable to mathematics students. It also includes ideas and discussions that will inspire the mathematics teacher to have more pride and confidence in his profession as well as to do a better job in the classroom. Although the applications that are presented are mostly of an advanced level they furnish good background material for mathematics teachers. In addition it attains one of its objectives by making the reader want to attend the Institute. This booklet is well worth its price.

EQUIPMENT

E. 33—Fraction Game

IMOUT (2 games); IMOUT CO.; P. O. Box 146, Station A, St. Petersburg, Florida

Price per set $3.65, plus $0.30 shipping charges

Description: This device, which gives practice in reduction of fractions, is an adaptation of the game of Bingo. On each of the forty-eight cards under each of the five letters I M O U T (I'm out) is a column of five reducible proper and improper fractions. Reduced fractions, whole numbers, and mixed numbers, which are equivalent to the unreduced fractions on the cards, are printed on small squares and one of the letters also appears on each square. The teacher can draw a square out of a box, read "*M*, 3/8" and every pupil will examine the five unreduced fractions in the *M* column of his card to see if he has 6/16 or some other equivalent of 3/8. Each pupil has red cardboard discs which he puts on any equivalent he may have found and the first pupil who gets five fractions covered vertically, horizontally, or diagonally wins the game providing he has no error. Included in the set is a tablet of master sheets for the teacher to use in keeping a record of fractions drawn. Also, a spinning device with the letters and fractions is included to provide a variation from drawing the small squares. Of the forty-eight cards provided, forty are for regular drill, and eight, which are slightly smaller, are for practice which is to be confined to either reduction of proper fractions or to reduction of improper fractions.

Appraisal: It is probably wise to limit the unreducible fractions on the small squares to denominations of twelve or smaller, but it is questionable that the reduced mixed numbers should be confined to those which always have a one in the unit's place. The equivalent proper and improper fractions on the IMOUT cards have a wide variety of denominators so a number of reductions are possible and considerable practice is thereby obtained. Also, much practice is further guaranteed because every pupil must evaluate five unreduced fractions every time the teacher calls a number. This device might actually provide more practice in reduction or recognition of equivalents than is needed by many of the pupils, but this could be controlled. The manual makes it clear that the device is only to be used as supplementary drill, so if this suggestion is heeded, the game is one good approach to drill in the skill emphasized. Pupils may have previously used comparable devices for practice with whole number operations, or in actually playing Bingo, so the mechanics of the game will probably not be troublesome.

The material is well constructed and carefully planned and the directions are adequate. (Reviewed by Robert L. Burch, Boston University, Boston, Mass.)

E.34—Multiplication and Division Game

Description: There are 48 IMOUT cards, each of which has a column of five multiplication or division combinations under each of the letters I M O U T—a total of twenty-five combinations on each

card. Twenty orange cards involve multiplication combinations for the 6 through 9 tables and twenty yellow cards contain division combinations which involve divisors or quotients from 6 to 9. The remaining 8 smaller sized cards have division and multiplication combinations which usually appear in the tables of 10's through 12's. The products or quotients are on small square pieces of cardboard. The teacher or a pupil can draw out one of these and call the letter and the answer shown. The pupils then cover the appropriate combination. In order to provide variation, there is a spinner with products and quotients which can be used instead of the small product or quotient squares. Printed master sheets are also provided so that the teacher can keep a record of answers that have been called and can check back when a pupil calls "I'm Out" after covering five combinations in a row, a column or on a diagonal.

Appraisal: This is a drill device and it is a good approach for teachers who are fully aware that drill is to be applied *after* understandings have been thoroughly developed. The plan of use requires that the pupil recognize rather quickly the answers for multiplication and division combinations, and provision is made for participation of every pupil. The concentration on the combinations which involve the larger numbers is wise because this device will be interesting enough so that pupils will not become bored quickly. In addition, they will get considerable practice on these hard combinations. Also, the IMOUT procedure can be adjusted easily to make it usable for small group work. (Reviewed by Robert L. Burch, School of Education, Boston University, Boston, Mass.)

INSTRUMENTS

I.22—Sextant

Edmund Salvage Company, P. O. Box 100, Audubon, New Jersey

Bubble Sextant; used government surplus; $12.50; Stock number 924

Description: This used army air force sextant is guaranteed to have been repaired and collimated to the satisfaction of the buyer. The sextant includes a flashlight rheostat for using the sextant at night, an auxiliary 2X Galilean telescope for use on faint stars, and a carrying case. The original cost of the sextant is given as about $125. For an additional $10, it is possible to purchase this sextant new with an electric automatic recording device and illuminated averaging disc for nighttime use. The instrument is sighted by manual operation and readings are registered automatically in one-second intervals on a marking disc enabling an average observation to be selected readily.

Appraisal: The sextant was designed for aerial naviagation by means of which the angular altitude of a heavenly body can be measured and recorded with reference to a bubble artificial horizon. It is a precision instrument that must be handled carefully to avoid damage to the point of making it inaccurate. Thus, it is quite fragile and complex for use in high school classes. However, it is indeed a bargain as long as it is available.

I.23—True Air Speed Computer

Edmund Salvage Company, P. O. Box 100, Audubon, New Jersey

Stock Number 937; 25 cents; War surplus

Description: This airspeed computer is an 8 inch circular scale similar to a circular sliderule. It can be used to compute the true air speed of an airplane, starting at 100 miles per hour, according to altitude and air temperature. It consists of two circular discs, one 8 inches in diameter the other 5½ inches, made from black plastic 1/10 inch thick. The original government cost was $2.20.

Appraisal: This inexpensive war surplus computer is worth its price to illustrate a simple application of mathematics to aviation. The principle of its operation would be appropriate supplementary material to present to a high school class working vector problems.

I.24—Drift Meter

Edmunds Salvage Company, P. O. Box 100, Audubon, New Jersey

Stock Number 942; $5.60; New war surplus

Description: This optical drift meter was used to determine the angle between the heading of an aircraft and its actual path over the ground. When the height of the aircraft is known, it can be used to determine the ground speed of the aircraft in knots or miles per hour. A Pantograph mechanism diagrams the drift of the plane to the left (plus) or to the right (minus). A computer, consisting of a logarithm scale, with height markings and a rotating seconds dial, is used to compute the speed of the aircraft by timing the passing of an image of a ground point from one line to the next.

Appraisal: This is another war surplus device that can be obtained at a small portion of its original price estimated to be $200. Although its application will be limited in high school mathematics classes, it is worth its price as an illustration of applied mathematics. It can be used to illustrate vector problems in aviation as well as to show the solution of problems by using proportions.

I.25—Torpedo Director

Edmund Salvage Company, P. O. Box 100, Audubon, New Jersey

Stock Number 934; $6.50; Used

Description: This instrument was used to direct the launching of torpedoes from aircraft. The instrument contains an achromat lens diameter 37 mms., and focal length 80 mms. in a threaded mount. The reflector plate is optically flat glass in the shape of an ellipse. It has a 28 volt light bulb in a reflector housing and a glass reticle with the aiming pattern engraved on it. It has a sun filter which may be used for the reflected aiming pattern. The instrument is packed in a sturdy wood case.

Appraisal: Although this device is only partially mathematical, it can be used in the measurement of angles in surveying. Another possible project would be the determination of how all the scales add up to the degree of the angle at which the torpedo is launched.

FILMS

F.52—The Meaning of (π) Pi

Coronet Films, Coronet Building, Chicago 1, Illinois

Educational Director, Harold Fawcett, Ohio State Univ. 1950, 16 mm. film 1 reel; Black and White $45.00; Color $90

Description: This film shows a boy at his desk doing mathematics problems about circles. It shows his notes and the papers on which he marks. The commentator then proceeds to explain all the uses of circles showing wheels, saws, round tables, etc. The boy then uses a phonograph record and yard stick to measure the diameter and circumference of a circle, while the commentator speaks. This is followed by schematic drawings of the same. The diameter length is marked on the yard stick and then the circumference, showing one to be about three times the other. The next scenes show the lengths of the diameters and circumferences of several circular objects and the boy proceeds to find the ratio of C to D. It closes with the definition of π and the boy's comments on the ease of understanding it. A short historical background of π is given showing the center of developments on a map.

Appraisal: The principal idea of the film is good but it moves rather slowly in the beginning and adds but little that the classroom teacher would not do. The film does not seem to show enough of the actual mathematical operation which is needed to point out true meaning. The vocabulary is good and the ideas are excellent.

I believe it would have value in the introduction of circles in grade seven and

eight and then used after the pupil has actually carried out the experiment himself. The film covers one topic and the attention was focused on it at all times. The last half of the film will hold the pupils' interest. It seems the technical aspects of the films were not as good as usual in that the speaking was not well done. (Reviewed by Philip Peak, Bloomington, Indiana)

F.53—The Language of Mathematics

Coronet Films, Coronet Building, Chicago 1, Illinois

Educational Director, Harold Fawcett, Ohio State Univ. 1950, 16 mm. film 1 reel; Black and White $45.00; Color $90.00

Description: This film attempts to show the need for the language of mathematics. The first scene shows a high school fire drill that was too slow. A committee is put to work and the teacher comments on the need for information in precise language. A series of running scenes of bridges, buildings, and Roman architecture are given, having superimposed on them the major topics of mathematics while the commentator describes the need for mathematical language. Many uses of numbers such as "how much," "how many," $E=Mc^2$, the slide rule, and the clock are shown. The fire drill committee reappears and each shows his findings in graphs or tables. The picture closes with a fire drill which is successful due to precise planning.

Appraisal: The entire film deals in broad generalities and consequently would be of little value as a teaching aid. No help is given as to how the language of mathematics enables one to get the desired results, only the results are shown. This is definitely a high school picture, but would do little toward motivation or appreciation of the language of mathematics for high school people. It is not thought provoking or entertaining. The material is all true and authentic, but aspects are fairly good. However the reviewer feels it will do little to increase understanding and appreciation of the language of mathematics. (Reviewed by Philip Peak, Bloomington, Indiana)

F.54—Multiplication is Easy

F.55—Division is Easy

Coronet Films, Coronet Building, Chicago, Illinois

Educational Director, F. Lynwood Wren, George Peabody College for Teachers, 16 mm. film 1 reel; each Black and White $45.00; Color $90.00; study guide

Description of F.54: This film secures real problems in multiplication by having Jack and Mary operate a Toy Rental Club in a garage. In order to determine the income from eight toys rented at five cents per week for three weeks, Jack uses multiplication. Mary solves the same problem by using addition. Using a different number of toys and weeks, Mary learns how to multiply, both from a table and by computation. The multiplication of 40 by 3 is used to illustrate the place value of numbers and how place value is taken into account in multiplication. Other business transactions give further practice in multiplication and emphasize that multiplying is easy.

Description of F.55: A Toy Rental Club operated by Jack and Mary in a home garage supplies problems for learning how to divide. For example, 96 cents is divided by 3 to find out how much can be spent for the purchase of three toys. Jack performs the division problems on a child's blackboard while Mary solves the problem by dividing the 96 cents into three piles of equal amounts. Jack shows Mary how a table of division facts can be used to shorten division. Coins are used to show the place value of figures and how place value is related to the division process. A problem in dividing 48 marbles into 3 equal sacks shows the meaning of remainder. Other real problems give further practice in division and emphasize that division is easy.

Appraisal of F.54 and F.55: The action in these films takes place in a natural setting with two children of the appropriate age playing the roles. The children are attractive and natural in their acting. The computational work is done on a child's blackboard. All the discussion is by the commentator although many of his words are the ones being spoken by the children. One wonders whether it would not be more effective to hear the actual conversation of the children. The commentary is very clear and appropriate for third or fourth grade presentation. The photography on the black and white print is very good and maintains interest. It is doubtful that the films will be more effective than the operation of a business or the manipulation of objects or money in the classroom. It will be valuable in showing teachers of arithmetic how to relate multiplication and addition, how to give meaning to division, how to emphasize place value, and how to set up real problems for learning and practice. The cheerful attitude of the performers toward the use of arithmetic may have a definite influence on the attitude of the pupils seeing the picture.

FILMSTRIPS

FS.72—The Circle

Photo and Sound Productions; 116 Natoma Street, San Francisco, California; Educational collaborator, O. W. McGuire; 1948; 35 mm. filmstrip; Black and White; Silent; 36 frames

Description: This is Part VI of the Study of Measurement series. Frames 3–8 define and illustrate circumference, diameter, radius, arc, sector and semicircle. The next diagram shows that two radii equal a diameter and gives the formula $2r=d$. The circumference of a circle is then pictured as a straight line with the diameter being contained in it three and a fraction times. The ratio of the circumference to the diameter is stated as 3.1416. The name *pi*, the symbol π and the value 3 1/7 follow. Another frame states these relationships between parts of a circle and its lines: $c=2r$; $c=\pi d$; $c/\pi=d$; $c/d=\pi$. All of these formulas are then used with a circle whose radius is 3. Frames 14 and 15 re-emphasize the circumference-diameter ratio. Frame 16 gives the area of a circle formula; 17 uses a square to show one meaning of "square a number"; 18 illustrates the area formula with a circle whose radius is 5. Frames 19–26 develop the area formulas $A=\frac{1}{2}cr$ and $A=\pi r^2$ by the method of dividing the circle into small sectors and transforming it into a figure approximating a rectangle. Frames 27–30 show solutions of the following types of examples: finding area when diameter is given; finding diameter when circumference is given; finding circumference when radius is given. The last two frames show that the area of a circular cylinder is equal to the area of a rectangle plus the areas of the two circular bases.

Appraisal: The only part of the filmstrip which would be an aid in developmental teaching is the section on the area approximating a rectangle. The drawings for this are not easily made at the blackboard. All the other information would be better taught and so better understood by having the students draw the figures and manipulate the objects. A filmstrip is not a good medium for working illustrative examples. Many teachers will object to the use of pi to four decimal places, a circumference of 18.8496 units and an area of 50.2656 square units with circles whose radii are given as 3 units and 4 units respectively. The material covered, the drawings, captions and vocabulary are junior high school level. Except as noted, the filmstrip has little to recommend it unless it be to furnish a new tack for reteaching or for review. (Reviewed by Frances Burns, Oneida High School, Oneida, New York.)

DEVICES FOR A MATHEMATICS LABORATORY

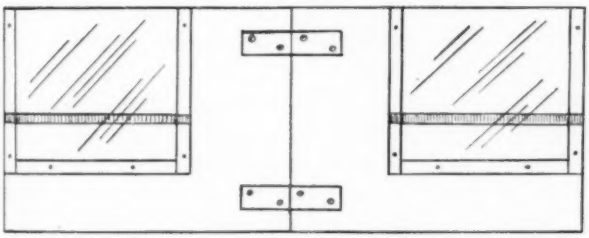
Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

THIS is the first of a series of articles describing mathematical devices which teachers and students can make for themselves. Student produced materials are often more helpful in facilitating understanding than commercially finished devices, because student construction in itself provides an exceptionally fine learning opportunity. It is with this thought in mind that THE MATHEMATICS TEACHER is introducing this special section as a regular feature. Since printed material on the production of learning aids is scarce, the success of this department will depend to a great extent on reader contributions. Anyone who has a favorite device which he would like to share with fellow teachers is invited to submit a drawing and description, similar to those below, to the department editor. Drawings sent in should be unmistakable in every detail and completely labeled, but they need not be drawn to any particular scale nor finished in any special way. Drawings submitted will be prepared for publication by this department. Send all materials to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

THE 45° ANGLE MIRROR*

The 45° angle mirror is a very useful instrument for solving practical problems in which it is necessary to construct a right angle. This instrument can be used to lay out circular tracks, to measure inaccessible distances, and to make maps by offsets. Other uses are discussed in *Field Work in Mathematics*, by Carl N. Shuster and Fred L. Bedford.[1]

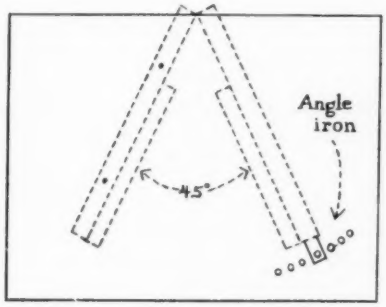
The construction of this instrument is simple and the materials needed are not difficult to acquire. Figures 1 and 2 illustrate the two separate parts of the completed instrument—a *Left Face* and a *Right Face* hinged together, and the *Base*. The *Left Face* is nailed to the *Base* as indicated in the diagram, but the *Right Face* swings free.



Left Face—
Nail to Base.

Right Face—
Swings Free.

FIG. 1



Top View of Base

FIG. 2

* Plans for the two devices described here were developed in the Mathematics Laboratory, Monroe High School, St. Paul, Minnesota. Drawings for printing were made by Patricia McGroder, Macalester College, St. Paul, Minnesota.

[1] Schuster, Carl N., and Bedford, Fred L., *Field Work in Mathematics*. New York: American Book Company, 1935, pp. 39–46.

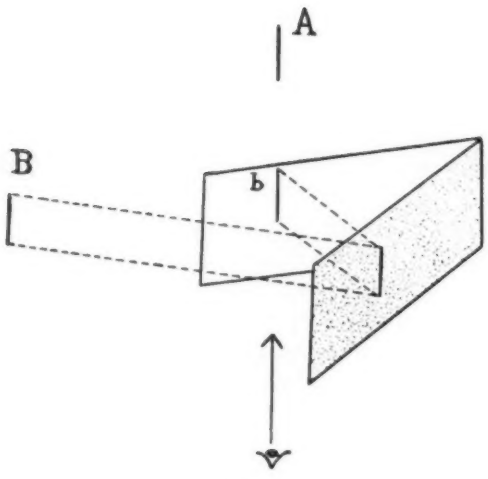


FIG. 3

Materials needed to make this instrument included the following: two pieces of wood for faces, $3'' \times 5'' \times \frac{1}{2}''$; one piece of wood for base, $5\frac{1}{2}'' \times 7'' \times \frac{1}{2}''$; twenty-six inches of wood for framing, $\frac{1}{4}'' \times \frac{1}{8}''$; two small butt hinges not over 2" in over-all width; one 90° angle iron, 1" arm length; two pocket mirrors, $3\frac{1}{4}'' \times 2\frac{1}{8}''$. Curtain hanger hooks can be substituted for wood framing. Using them will save a lot of work. Also, one long narrow hinge can be substituted for the two smaller ones listed above.

The diagrams of Figures 1 and 2 show how the materials are assembled. The stripping around the mirror is used to hold the mirror in place. One strip across the front of each mirror (the shaded strip) will hold it firmly against the back of the face. If necessary, blotter paper may be used between the mirror and the wooden face to tighten the mirror securely. Fastening the mirrors with glue is not recommended because this procedure usually destroys the silvering.

After the instrument has been assembled, the 45° angle should be determined as accurately as possible. The faces may be opened to 45° with the aid of a protractor but it is more satisfactory to make this adjustment by using a known right triangle. In following the latter suggestion, the instrument should be placed at the right angle vertex and the swinging face (*Right Face*) moved "into position." If in Figure 3 we assume that the instrument is at the right angle vertex of the triangle, and that A and B are stakes at the ends of the hypotenuse, then the 45° angle is determined if we adjust the shaded or *Right Face* so that a reflection of B on the fixed or *Left Face*, say b, is in line with the observer and the fixed A. A thumb tack can now be used to secure the swinging face to the base.

A right angle can always be located if any two arbitrary points such as A and B are taken as the ends of an hypotenuse. The instrument itself must be moved around until one of the stakes A can be seen over the top of the image b of the other stake. The instrument is then at the right angle vertex of the right triangle for which AB is the hypotenuse.

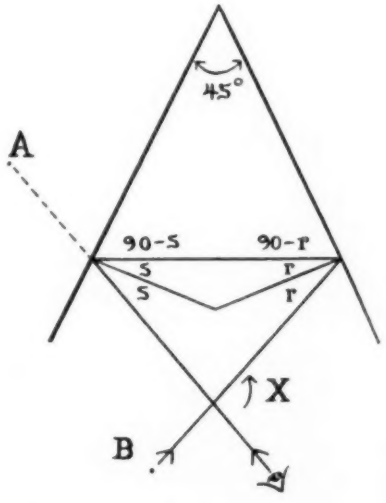


FIG. 4

Following is a proof of the instrument:

See Figure 4. Assume that the angle between the faces is 45°.

$X = 2r + 2s$.

$(90 - r) + (90 - s) + 45 = 180$.

$r + s = 45$.

$X = 2(r + s)$.

$X = 90$.

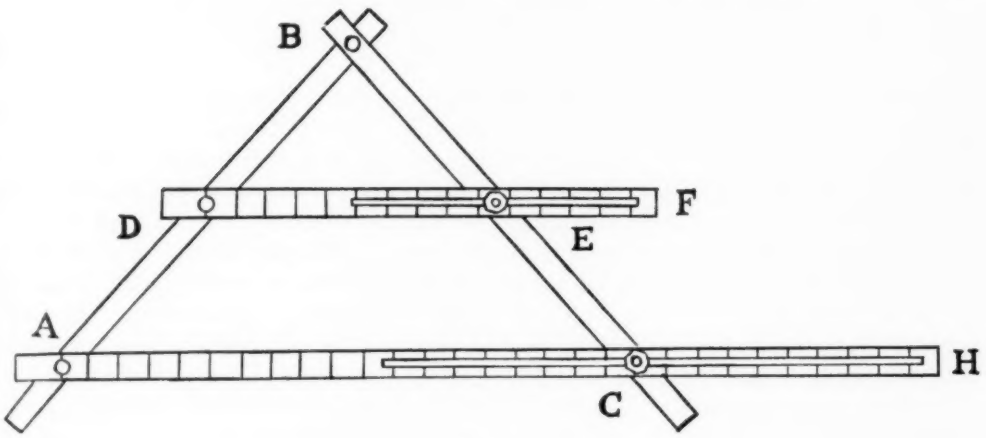


FIG. 5

A THEOREM DEMONSTRATION DEVICE

The theorem demonstration device shown in Figure 5 can be constructed very easily and will help give meaning to the theorem, "A line joining the midpoints of two sides of a triangle is parallel to the third side and equal to one-half of it."

Materials needed to produce the device include four strips of thin board or cardboard, $\frac{3}{4}''$ in width; five metal paper rivets or $\frac{1}{8}''$ stove bolts, and seven metal washers which fit the rivets or bolts. The illustration in Figure 5 shows rivets at points A, D, and B, and bolts at E and C.

The dimensions of the device shown in the illustration are as follows: $AH = 14''$, $AB = 6''$, $BC = 8''$, and $DF = 7''$. Note that $AD = DB$, and $BE = EC$. In cutting the strips from wood or cardboard approximately one inch should be added to each of the above dimensions, so that the holes can be punched or drilled at the proper places. The device is assembled as illustrated. Slits in DF and AH must be wide enough to permit easy passage along the bolts at E and C. Strips DF and AH should be marked off in half inches or centimeters from D and A respectively so that the meaning of the theorem can be demonstrated concretely.

The size of the device can be enlarged by doubling or tripling the dimensions listed above.

Careers with a Future—The Public Health Statistician

The American Public Health Association has recently put forth a pamphlet which is one of a series concerned with careers with a future. It is entitled "The Public Health Statistician," and is directed at acquainting the undergraduate and graduate student with the opportunities in the field of public health statistics. Since a number of the present workers in public health statistics were originally trained in mathematics and have found that public health has given them an opportunity for application of their mathematical knowledge, the pamphlet should be of interest to those who advise mathematical students in their search for job opportunities. The pamphlet discusses the work of the public health statistician, the educational requirements for this work, employment opportunities, and compensation. Copies may be obtained, in limited numbers, by writing to the American Public Health Association, 1790 Broadway, New York 19, New York.

RECREATIONAL MATHEMATICS

Edited by AARON BAKST

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THIS is the beginning of a new department in THE MATHEMATICS TEACHER. This department has a purpose. Its aim is to assist the classroom teacher in putting color and life in everyday teaching. There are many ways and means how this might be achieved. Generally, recreations are supposed to introduce elements of interest and motivation. On the other hand, recreations, as they have been known in the mathematical literature for centuries, have been centered around the puzzling and the play with mathematical operations. This may be interesting, but only for a while. Soon the interest in such things may wear off. This acts as a warning that we should not become too enthusiastic over such types of recreations. If we teach mathematics from such recreational points of view only, we may obscure the more important aims of the mathematical instruction. Such recreations are like sweets. Too many sweets may be more harmful than no sweets at all. But when administered in moderation and with variation, sweets may have a very beneficial effect. And so it is with recreations. The everyday work in many mathematics classrooms may soon begin to seem monotonous. Again here we can draw an analogy. Modern dietetics strongly recommends variation in eating habits. In teaching, the same principle holds. So, recreations, applications, illustrations, all these practices, and many others of similar nature should serve the purposes of varying the mental food which is placed before the pupils who study mathematics.

Recreations have the properties akin to those of Vitamin D, the sunshine vitamin. They break the monotony of the dryness of mathematical instruction. Mathematics may appear very dry and humorless to the pupil. Recreations may evoke the smile which will break the ice so often pervading the atmosphere in a mathematics classroom. A teacher should never shun the opportunity to introduce a humorous situation, even to the extent of making it seem to be ridiculous, provided the opportunity is present and the illustration is appropriate. It is not *what we teach* that is important. The important thing is *the way we teach*. The same topic may be presented in a purely abstract form, and it will fall flat. Dress up this topic in something which has a human touch in it, and it becomes a huge success. This is the aim of this department. It is hoped that the classroom teacher will find in this department the material which will assist him (or her) in planning a lesson which will enthrall the pupils, even those who are unresponsive to mathematics.

The columns of this department are presented to the classroom teacher as a service. The material which, we hope, will appear here regularly will be definitely restricted to the subject matter which is taught in the elementary and secondary schools. The recreational material will be presented so that the teacher will be able to present it in the actual classroom situation without interrupting the regular teaching procedure. As much as space will permit, an analysis of the mathematical topics which are involved in the presentation of this material will accompany the illustration. Hints for variations and elaborations will be also stated.

One more word concerning recreations in mathematical instruction. Traditionally, recreations are associated with puzzles, tricks, purely mathematical problems, and so on. We think that recreations need not be dressed up in a purely mathematical robe. They may represent problems which are so worded that they represent something which has a human element, but at the same time they border on the puzzling.

They may be problems which are cryptic in form. They may be interesting constructions, they may be puzzling graphs and charts. The readers of THE MATHEMATICS TEACHER will find in this column references to sources which contain such material.

Since this is a department which, we believe, should interest many teachers, the active participation of the readers of THE MATHEMATICS TEACHER is invited and urged. If you have any interesting material which might be used, please send it in. If you have any specific questions and problems which you would like to be discussed, please write us. Not all can be answered in print. But you will receive a prompt reply, provided you give your return address. Address all communications concerning the material in this department to: Dr. A. Bakst, Department of Mathematics, School of Education, New York University, Washington Square, New York 3, N. Y.

Incidentally, we have a Christmas greeting prepared for you. It will appear in the November issue of THE MATHEMATICS TEACHER.

As an example of the kind of material this department will feature and analyze let us consider the following puzzler: *Write any positive integer by means of three twos and a combination of mathematical symbols.*

Let us illustrate this problem by solving a specific case. Let the given positive integer be 3. Then

$$3 = -\log_2 \log_2 \sqrt{\sqrt{\sqrt{2}}}.$$

We can convince ourselves of the correctness of the solution as follows:

$$\sqrt{\sqrt{\sqrt{2}}} = [(2^{1/2})^{1/2}]^{1/2} = 2^{1/8}$$

Then

$$\log_2 2^{1/8} = \tfrac{1}{8},$$

and

$$-\log_2 \log_2 2 = -\log_2 \tfrac{1}{8} = -\log_2 2^{-3}$$
$$= -(-3) = 3.$$

If the positive integer were 5, then the solution would be

$$5 = -\log_2 \log_2 \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}.$$

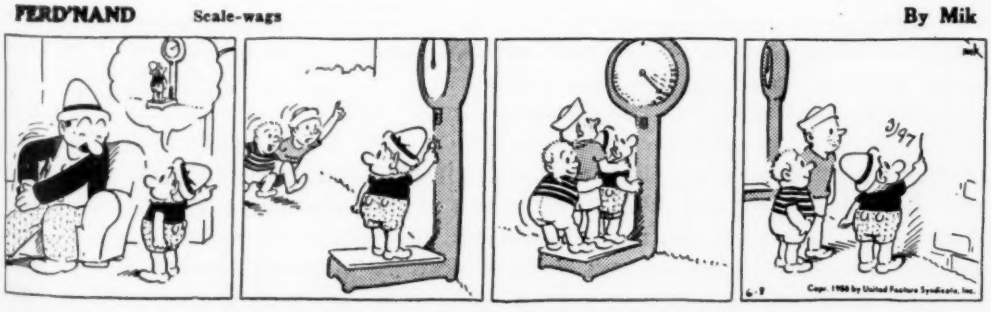
For any positive integer n the solution is

$$n = -\log_2 \log_2 \sqrt{\sqrt{\cdots\sqrt{\sqrt{2}}}}.$$

(n radicals)

A problem of this type is suitable for intermediate algebra classes whenever work with radicals (or fractional exponents) is presented in conjunction with the topic in logarithms.

As an interesting exercise would be the question of writing any positive integer with a certain number of 3's and mathematical symbols, or a certain number of 4's and mathematical symbols. The solution of such a recreation problem will be discussed in a later issue of THE MATHEMATICS TEACHER.



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NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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ONE of the difficulties we meet in teaching mathematics is the conviction which many people have that mathematics is a finished body of knowledge. Our pupils tend to take for granted that mathematics has always been as it now is and that it will always remain in that state. The use of materials from the history of mathematics helps to meet this situation, and class discussions become more interesting when it is realized that mathematical concepts, notations, and processes are the product of an evolution that is not yet complete, and that will probably continue to advance as our civilization becomes even more complex. Students are annoyed when they are introduced to fractional exponents when radical signs seem an adequate way to indicate roots. Why not choose one method and stay with it? It is a matter of surprise to discover that today 3.50 means the product of 3 and 50 in England and that the number an American writes as 3,500 would be written as 3.500 by a Frenchman. Apparently even ordinary notations are not entirely international. The topic of weights and measures becomes much more alive when a class discusses the social and economic importance of standardized units and then brings the subject down to earth by investigating the ways in which people have coped with this problem.

The purpose of a department of THE MATHEMATICS TEACHER devoted to the history of mathematics is to provide a series of short resource units that are particularly adapted to the enrichment of the mathematics work from the elementary school through the junior college and the teachers college. Ideally, each unit should include references to more detailed discussions for the convenience of those who wish to follow it further. This department will fail in its purpose if it becomes merely a scrap book of possible suggestions. It will succeed in its purpose to the extent to which it stimulates the reader to launch out for himself and do independent work along these lines.

This department will not attempt to present technical researches. They belong elsewhere. On the other hand, it should avoid oversimplification and the habit of "romancing," of expressing the thoughts you think you would have had, had you been hit by Isaac Newton's mythical apple or if you had been Descartes watching the legendary spider. It should be possible, however, to present materials in a way that is in accord with the facts as we know them, in a way that is academically respectable, in a way that is direct and simple but that will not indulge too much in the process that Sir Charles Singer describes as "making a complex truth into a simple falsehood."

No attempt will be made to tell where and how these materials may be used. That is an individual problem. It depends on the teacher, the students, and the situation.

It would be helpful if suggestions as to topics for discussion were to be sent to the editor of this department of THE MATHEMATICS TEACHER. And, if this department proves of significant value, it would be interesting at some time to have a symposium of the ways in which the history of mathematics is being used under various conditions.

THE ART OF RECKONING

INTRODUCTION

In 1937, the National Council of Teachers of Mathematics published a pamphlet by Professor D. E. Smith and Professor Jekuthiel Ginsburg on *Numbers and Numerals*. This proved so helpful that it has been reprinted and is again available.

The usefulness of this pamphlet suggested that its hints on how people have used numerals in computation should be expanded into an account of everyday reckoning assembling the material from the actual aids to computation that are still extant, from references in literature, and from textbooks especially those devoted chiefly to arithmetic.

In this discussion, it must be remembered that the topic is computation in the hands of ordinary folk, not computation as it was done by scholars; not the few who had access to the knowledge that the universities could offer, but those whose schooling was meagre if in fact they had had any formal training whatever. Under these circumstances, it will be realized that ease of mastery of a topic is so important that people tend to continue to use a time-consuming method of computation rather than to learn a more complicated method which ultimately is the quicker.

It must be remembered that there were wide diversities of practice even among contemporaries. Arithmetic with counters and arithmetic with the pen existed side by side for generations. Ideas had to work slowly into popular use. Because of the diversities between countries and between groups within a single country, it is impossible to develop the story of computation in a chronological way. Instead it is convenient to begin with the simplest method of computation, that is with tallies, and then to consider counter reckoning, and finally ciphering, that is computation using Hindu-Arabic numerals.

I. Tallies

Computing without Counting, Recording Data without Numerals

The use of notches cut in a piece of wood, knots tied in a cord, or lines drawn on any convenient surface are methods recording data that may conveniently be grouped under the name of *tallies*. The method is a simple one, wide-spread both in time and geographically. In the Egyptian collection in the Metropolitan Museum of Art in New York City there is a foreman's tally of the work done by his men. It is a piece of stone with marks scratched on it. Robinson Crusoe used this scheme when he daily made a cut in his calendar stick. We use pencil marks in groups of five to keep score in a game.

In the Book of Ezekiel, the prophet tells of a man "with a line of flax in his hand and a measuring reed." The reed is definitely stated to be a measuring instrument and the line of flax seems to have been a sort of field note book where knots showed the record.

At the time of the Spanish conquest, the Incas in Peru had a well-developed system of keeping records by knotted cords called *quipu*. There is a story that knotted cords had an important part in a revolt against the Spaniards. The instigators of the revolt fixed a day when it was to occur and sent messengers to the villages involved. Each messenger carried a knotted cord and untied a knot each day. The revolt took place on the day when the last knot was untied.

There is an account of a census taken in India in 1872 where cords of different colors represented different categories and knots recorded the count of individuals in each group. The census taker had no need of reading or writing.

It will be noted that the value of these records lies in having a one-to-one correspondence between the objects and the knots or marks. If there is a mark for each object and an object for each mark, there will be exactly as many marks as objects. Records such as these may be sufficient to compare things of two groups as for example to tell whose score is the greater in a game. It may also be a substitute for computation as was true in the case of a certain workman. He was unable to read or write but he always carried a piece of chalk in his pocket and every time the town clock struck while he was working he made a mark on the handle of his wheelbarrow. At the end of the day, he collected an hour's wages for each mark.

Tallies have also been used as a legal device for keeping records. In the Middle Ages, tally sticks constituted a record of money that had been borrowed—a promissory note if you will. Notches were cut cross-wise of a flat stick, the type and depth of the cut indicating a specific amount of money—a pound, a shilling, a penny for instance. The stick was then split lengthwise so that the notches appeared on both pieces. The debtor received the smaller part which in England was called the *foil,* and the creditor kept the larger part called the *stock*. A memorandum of the transaction giving names and dates was sometimes written along the side of the stick. When the note was paid, the two parts of the stick were matched against each other. Then the two parts of the tally were broken and the note was cancelled. "Making accounts tally" was a physical process. Since the person who loaned the money kept the *stock* of the tally stick, the word *stockholder* was applied to a person who had invested capital in a company and his investment received the name of *stock*.

The Court of the Exchequer in England made extensive use of tallysticks as receipts for payments made to the government. The practice was a convenient one before the invention of paper for it provided a simple, cheap, and tamper-proof record. But the use of tally sticks continued almost to the nineteenth century which was perhaps beyond the time of their real usefulness. In 1834, the tallies having been obsolete as receipts for better than a generation, the tally sticks that had accumulated in the attics of the Houses of Parliament were ordered to be destroyed. The burning tallies overheated the chimneys in the Houses of Parliament, the Parliament Buildings caught fire and the tallies and the buildings went out of existence simultaneously. The use of tallies for keeping records, however, has continued in various ways and in many parts of the world.

Allied to the use of tallies for records is the use of various types of tallies in the arithmetic of the early grades where these methods of representing quantities are an important aid in the determination of sums and differences.

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TOPICS OF INTEREST TO MATHEMATICS TEACHERS

Edited by William L. Schaaf

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Mathematics and Modern Science

As I have pointed out elsewhere, the ideal professional equipment of the teacher of mathematics does not cease with reasonable mastery of the fundamental branches of mathematics and a wide familiarity with educational theory and practice. There is yet a third area, equally as important as these two: an adequate background of appreciation of mathematics. By this is meant some insight into the nature of mathematics, and its historical development; into the relations of mathematics to logic and philosophy; and into the broader cultural aspects of mathematics, such as its relation to pure science, to technology, to the fine arts, to the nature of thinking, and to the humanities.

This appreciation is not always readily attainable. The relevant literature is for the most part scattered and diffuse. The

same may be said, although to a lesser degree, for the literature of mathematical education.

It would seem, therefore, that this Department of THE MATHEMATICS TEACHER, which makes its initial appearance herewith, might well prove useful to teachers who wish to extend their appreciational background of pure and applied mathematics, as well as to keep up to date on purely professional matters. At least that is our aim, and while we shall endeavor to select topics of likely interest to teachers, we cordially invite your requests for bibliographic material on specific topics of interest to you, In general, our attention will be divided between topics in mathematics *per se* and topics in the area of mathematical education. Requests have already been received from some of our readers for material on the relation of mathematics to pure science.

The changes that have taken place in modern science in the last forty years are at once breath-taking and spiritually uplifting. The rôle played by mathematics in this drama is unique. The story cannot be told in a few words. The literature is enormous; selection is both imperative and difficult. While some of the references given below are of a "popular" nature, for the most part the discussions are fairly technical. Although brief, it is hoped that this list will prove a helpful guide.

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Whittaker, E. T. "Chance, freewill and necessity in the scientific conception of the universe." *Physical Society of London, Proceedings*, 1943, 55: 459–471.

Wisdom, J. O. "Philosophical foundations of quantum mechanics." *Mind*, 1947, 56: 77–81.

Wolff, Georg. "Mathematics as related to other great fields of Knowledge. Part I: Mathematics and the Sciences." *National Council of Teachers of Mathematics, Eleventh Yearbook*, 1936, pp. 207–225.

BOOK SECTION

THIS section presents the latest books which have been received for review in THE MATHEMATICS TEACHER. Reviews of many of these books will appear in the monthly issues. Members of the Council are invited to send us further comments or corrections of errors relating to any of the books mentioned. In addition, a free loan service is being arranged whereby any member may borrow any of the books listed for a period not to exceed two weeks. Requests should be addressed to THE MATHEMATICS TEACHER, 212 Lunt Building, Northwestern University, Evanston, Illinois.

BOOKS RECEIVED

ELEMENTARY SCHOOL

1. Texts

Numbers We See, by Anita Riess, Maurice Hartung, and Catherine Mahoney. First-grade number-readiness book. Teacher's Edition, 162 pages, with sample oak-tag window and frame, cloth, 1948. Scott, Foresman and Company, 433 E. Erie St., Chicago 11, Ill., $1.32.

Arithmetic 3: The World of Numbers, by Dale Carpenter, Supervisor of Mathematics, Los Angeles City Schools; and Esther J. Swenson, Professor of Elementary Education, University of Alabama. Cloth, 316 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

Arithmetic 4: The World of Numbers, by Dale Carpenter, Supervisor of Mathematics, Los Angeles City Schools; and Lester G. Anderson, Dean, Teacher Education, Colleges of The City of New York. Cloth, 316 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $1.68.

2. Workbooks

Growing Up With Numbers, Books I, II, III, by Rose and Ruth Weber. Paper, 96 pages, 112 pages, and 112 pages respectively, 1942. McCormick-Mathers Publishing Company, P.O. Box 2212, Wichita 1, Kans. $0.27, $0.30, $0.30.

Growing Up With Arithmetic, Books IV, V, VI, by Rose and Ruth Weber. Paper. 128 pages, 144 pages, 144 pages respectively, 1942, 1943, 1946. McCormick-Mathers Publishing Company P.O. Box 2212, Wichita 1, Kans. Achievement tests provided with workbook. $0.36, $0.39, $0.39.

Making Arithmetic Plain, Books I and II, by Rose and Ruth Weber. Paper, both 144 pages, 1949. McCormick-Mathers Publishing Company, P.O. Box 2212, Wichita 1, Kans. Achievement tests provided with workbook. $0.39.

3. Supplementary Materials

Arithmetic Readiness Cards, Set I: Grouping, by Maurice L. Hartung, Henry Van Engen, Helen Palmer. 54 cards with 9 sheets of number cutouts and teacher Guide-book, 1949. Scott, Foresman and Company, 433 E. Erie St., Chicago 11, Ill., $3.20.

HIGH SCHOOL

1. General Mathematics

Everyday General Mathematics, Book One, by William Betz, Specialist in Mathematics, Rochester, New York; A. Brown Miller, West Technical High School, Cleveland, Ohio; F. Brooks Miller, formerly, Shaker Junior High School, Shaker Heights, Ohio; Elizabeth B. Mitchell, Charlotte High School, Rochester, New York; and H. Carlisle Taylor, Head of the Department of Mathematics, Benjamin Franklin High School, Rochester, New York. Cloth, xii+436 pages, 1949. Ginn and Company, 72 Fifth Avenue, New York, N. Y. $2.40.

Making Mathematics Work, by Gilbert D. Nelson, Chairman of Mathematics Department, Lincoln High School, Cleveland, Ohio; and Herschel E. Grime, Directing Supervisor of Mathematics, Cleveland Public Schools. Cloth, ix+630 pages, 1950. Houghton-Mifflin Company, 2 Park Street, Boston 7, Massachusetts. $2.40.

Mathematics in Daily Use, Revised Ed., by Walter W. Hart, author of mathematics texts; Cottell Gregory, Louisville Girls High School, Louisville, Kentucky; and Veryl Schult, Supervisor of Mathematics, Divisions I–IX, Washington, D. C. Cloth, vii+376 pages, 1950. D. C. Heath and Company, 285 Columbus Ave., Boston 16, Mass. $2.04.

2. Algebra

Teachers Manual, to accompany *Algebra, Book I*, by A. M. Welchons and W. R. Krickenberger, the Arsenal Technical High School, Indianapolis, Indiana. Paper, ix+27 pages, 1949. Ginn and Company, 72 Fifth Avenue, New York, N. Y. $0.48.

3. Plane Geometry

Plane Geometry, by Walter W. Hart, formerly Associate Professor of Mathematics, School of Education, University of Wisconsin. Cloth, ix+382 pages, 1950. D. C. Heath and Company, 285 Columbus Ave., Boston, Massachusetts. $2.00.

Plane Geometry, by William G. Shute, William W. Shirk, and George F. Porter, Instructors in Mathematics, The Choate School, Wallingford, Conn. Cloth, viii+406 pages, 1949. American Book Company, 88 Lexington Avenue, New York 16, N. Y. $2.48.

4. Second Course in Algebra

Algebra, Book Two, by A. M. Welchons and W. R. Krickenberger, the Arsenal Technical High School, Indianapolis, Indiana. Cloth,

x+516 pages, 1949. Ginn and Company, 72 Fifth Avenue, New York, N. Y. $2.20.

A Second Course in Algebra, Second Rev., by N. J. Lennes and J. W. Maucker, Dean, School of Education, Montana State University. Cloth, xv+522 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, New York, $2.48.

Second Year Algebra: New Edition, by Raleigh Schorling, Head, Department of Mathematics, The University High School, and Professor of Education, University of Michigan; Rolland R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts; and John R. Clark, Teachers College, Columbia University. Cloth, xii+500 pages, 1950. World Book Company, 313 Park Hill Avenue, Yonkers-on-Hudson, New York. $2.20.

5. Special Courses in Mathematics

Arithmetic in Agriculture, by Theodore H. Fenske, Associate Director of Field Operations, Department of Agriculture, University of Minnesota; Richard M. Drake, Professor of Education, University of Buffalo; Allen W. Edson, Assistant Professor, West Central Schools and Experiment Station, Morris, Minn.; and A. M. Field, Editor, Professor of Agricultural Education, University of Minnesota. Cloth, x+243 pages, 1950. The Webb Publishing Company, 55 E. 10th St., St. Paul, Minnesota. $1.60.

Cotton Mill Mathematics, by Thos. H. Quigley, Georgia School of Technology, Atlanta; and W. S. Smith, Overseer of Weaving, Pacolet Mill, New Holland, Ga. Cloth, xiv+322 pages, 1930. Turner E. Smith and Company, 441 W. Peachtree Street, N.E., Atlanta, Ga. $2.67.

Practical Mathematics, Part I, Arithmetic with Applications, 5th Edition by Claude Irwin Palmer, late Professor of Mathematics and Dean of Students, Armour Institute of Technology; and Samuel Fletcher Bibb, Associate Professor of Mathematics, Illinois Institute of Technology. Cloth, xii+179 pages, 1949. McGraw-Hill Book Company, Inc., 330 W. 4nd St., New York 18, N. Y. $1.80.

Practical Mathematics, Part II, Algebra with Applications, 5th ed. by Claude Irwin Palmer, late Professor of Mathematics and Dean of Students, Armour Institute of Technology; and Samuel Fletcher Bibb, Associate Professor of Mathematics, Illinois Institute of Technology. Cloth, xiii+252 pages, 1950. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York. $2.20.

6. Workbooks, Supplementary Materials and Tests

Self-Help General Mathematics Workbook, by G. E. Hawkins, Chairman, Department of Mathematics, Lyons Township High School and Junior College, La Grange, Ill.; and L. S. Walker, Head of Mathematics Department, Niles High School, Niles, Michigan. Contains thirty standardized, cumulative self-testing drills and five self-help study units on general mathematical concepts. 80 pages, paper, 1949. Scott, Foresman and Company, 433 E. Erie St. Chicago 11, Illinois. $0.72.

Workbook in Geometry, by Virgil S. Mallory, Professor of Mathematics and Instructor in Geometry, State Teachers College, Montclair, N. J.; and Zaidee T. Horsfall, Registrar and teacher of Mathematics, Queen Elizabeth High School, Halifax, Nova Scotia. Paper, 176 pages, 1949. Benj. H. Sanborn and Company, 221 E. 20th Street, Chicago. $1.00.

Plane Geometry Experiments, by Allene Archer, Thomas Jefferson High School, Richmond, Virginia; Miles C. Hartley, Assistant Professor of Mathematics, College of Education, University of Illinois; and Veryl Schult, Head of Department of Mathematics, Public Schools, (Div. 1–9), Washington, D. C. Paper, vi+78 pages with 20 plates, 1949. D. Van Nostrand Company, Inc., 250 4th Avenue, New York, $0.96.

Instructional Tests in Plane Geometry, by Florence C. Bishop, Chairman Mathematics Department, Central High School, Flint, Michigan; and Manley E. Irwin, Supervising Director of Instruction, Public Schools, Detroit, Michigan. Paper, x+67 pages, 1950. World Book Company, 313 Park Hill Ave., Yonkers-on-Hudson, New York. $0.56.

College

1. 1st Year Mathematics

Basic Mathematics for General Education, by H. C. Trimble, F. C. Bolser, and T. L. Wade, Jr., of the Mathematics Department, Florida State University. Cloth, xiii+313 pages, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. $3.25.

Elements of Mathematical Analysis, by Samuel E. Urner, Los Angeles City College; and William B. Orange, late of Los Angeles City College. Cloth, xi+561 pages, 1950. Ginn and Company, 72 Fifth Avenue, New York. $4.00.

Primer of College Mathematics, by John F. Randolph, Professor and Chairman of the Department of Mathematics, University of Rochester. Cloth xii+545 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $4.75.

2. Trigonometry

Plane Trigonometry, by John J. Corliss and Winifred V. Berglund of the Chicago Undergraduate Division, University of Illinois. Cloth, xii+388 pages, 1950. Houghton-Mifflin Company, 2 Park Street, Boston 7, Mass. $3.00.

Plane Trigonometry, Alternate Edition, by E. Richard Heineman, Professor of Mathematics, Texas Technological College. Cloth, xiv+184 pages and tables, 1950. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York. $2.50.

Plane and Spherical Trigonometry, Revised Edition, by John A. Northcott, Professor of Mathematics, Columbia University. Cloth, ix+234 pages with tables, 1950. Rinehart and

Company, Inc., 232 Madison Avenue, New York. $3.50.

Plane and Spherical Trigonometry, Fifth Edition, by Claude Irwin Palmer, late Professor of Mathematics and Dean of Students, Illinois Institute of Technology; Charles Wilbur Leigh, late Professor of Analytic Mechanics, Illinois Institute of Technology; and Spofford Harris Kimball, Professor of Mathematics and Head of the Department of Mathematics and Astronomy, University of Maine. Cloth, xiii+265 pages and tables, 1950. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York, $3.25.

Plane and Spherical Trigonometry, Third Edition, by H. L. Rietz, late Professor of Mathematics, University of Iowa; J. F. Reilly, late Professor of Mathematics, University of Iowa; and Roscoe Woods, Associate Professor of Mathematics, University of Iowa. Cloth, xiii+204 and tables, 1950. The Macmillan Company, 60 Fifth Avenue, New York. $3.00. Without tables, $2.75.

Plane and Spherical Trigonometry, by M. Richardson, Assistant Professor of Mathematics, Brooklyn College. Cloth, xiv+323 pages with tables, 1950. The Macmillan Company, 60 Fifth Avenue, New York. $3.75, without tables, $3.40.

Essentials of Plane Trigonometry, with Tables, by Joseph B. Rosenbach, Edwin A. Whitman, and David Moskovitz, of Carnegie Institute of Technology. Cloth, vii+158 pages, 1950. Ginn and Company, Statler Building, Boston. $2.70.

3. Algebra

College Algebra, by Harry A. Bender, Associate Professor of Mathematics, Rhode Island State College. Cloth, xi+449 pages, 1950. Pitman Publishing Corporation, 2 West 45th Street, New York. $3.50.

College Algebra, by Earle B. Miller, Professor of Mathematics, Illinois College; and Robert M. Thrall, Associate Professor of Mathematics, University of Michigan. Cloth, xvii+493 pages, 1950. The Ronald Press Company, 15 E. 26th Street, New York. $3.75.

Commercial Algebra, College Course, by Thomas Marshall Simpson, Head of Mathematics Department and Dean of Graduate School, University of Florida; Zareh M. Pirenian, Associate Professor of Mathematics, University of Florida; and Bolling H. Crenshaw, late Head of Mathematics Department, Alabama Polytechnic Institute. Cloth, xi+173 pages, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York. $2.50.

Fundamental Algebra with Practical Applications, by Robert L. Erickson, Professor of Mathematics, Lebanon Valley College. Cloth, xi+317 pages, 1949. McGraw-Hill Book Company, Inc., 33 W. 42nd Street, New York. $2.80.

Intermediate College Algebra, by Edward M. J. Pease, Professor of Mathematics and Electrical Engineering, Mathematics Department Head, Rhode Island State College. Cloth, vii+420 pages, 1950. Prentice-Hall, Inc., 70 Fifth Avenue, New York. $2.85.

4. Analytic Geometry

Analytic Geometry. by Raymond D. Douglass, Professor of Mathematics, Massachusetts Institute of Technology; and Samuel D. Zeldin, Associate Professor of Mathematics, Massachusetts Institute of Technology. Cloth, ix+216 pages, 1950. McGraw-Hill Book Company Inc., 330 W. 42nd Street, New York. $2.75.

Analytic Geometry and Calculus, by Harold J. Gay, late Professor of Mathematics, Worcester Polytechnic Institute; edited by Raymond K. Morley, Professor of Mathematics, Worcester Polytechnic Institute. Cloth, vii+524 pages, 1950. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York. $5.00.

Analytic Geometry, by Lyman M. Kells, Professor of Mathematics, United States Naval Academy; and Herman C. Stotz, Associate Professor of Mathematics, United States Naval Academy. Cloth, viii+280 pages, 1949. Prentice-Hall, Inc., 70 Fifth Avenue, New York. $2.85.

Elements of Analytic Geometry, Third Edition, by Clyde E. Love, Professor Emeritus of Mathematics, University of Michigan. Cloth, xii+218 pages, 1950. The Macmillan Company, 60 Fifth Avenue, New York. $2.75.

Analytic Geometry, Third Edition, by W. A. Wilson, late Professor of Mathematics, Yale University; and J. I. Tracey, Associate Professor of Mathematics, Yale University. Cloth, x+318 pages, 1949. D. C. Heath and Company, 285 Columbus Avenue, Boston. $2.75.

5. Statistics

Experimental Designs, by William G. Cochran, Professor of Biostatistics, The Johns Hopkins University; and Gertrude M. Cox, Director, Institute of Statistics, University of North Carolina. Cloth, ix+454 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York. $5.75.

Some Theory of Sampling, by William Edwards Deming, Adviser in Sampling, Bureau of the Budget, Washington, Adjunct Professor of Statistics, Graduate School of Business Administration, New York University. Cloth, xvii+602 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York. $9.00.

An Introduction to Probability Theory and Its Applications, by William Feller, Professor of Mathematics, Cornell University. Cloth, xii+419 pages, 1950. John Wiley and Sons, Inc., 440 Fourth Avenue, New York. $6.00.

6. Advanced Mathematics

The Anatomy of Mathematics, by R. B. Kershner, Applied Physics Laboratory, Johns Hopkins University; and L. R. Wilcox, Associate Professor of Mathematics, Illinois Institute of Technology. Cloth, xi+416 pages, 1950.

Ronald Press, 15 E. 26th Street, New York. $6.00.

Differential Algebra, by Joseph Fels Ritt, Davies Professor of Mathematics, Columbia University. Cloth, viii+184 pages, 1950. American Mathematical Society, 531 W. 116th Street, New York. $4.40.

Geometrical Tools, A Mathematical Sketch and Model Book, by Robert C. Yates, U.S.M.A. Paper, 194 pages, 1949. Educational Publishers, Inc., St. Louis. $3.00.

Numerical Solutions of Differential Equations, by H. Levy, Professor of Mathematics, Imperial College of Science, University of London; and E. A. Baggott, Lecturer in Mathematics, The Polytechnic, London. Cloth, viii+238 pages, 1950. Dover Publications, Inc., 1780 Broadway, New York. $3.00.

Theory of Sets, by E. Kamke, Professor of Mathematics, University of Tubingen; trans. F. Bagemihl, Assistant Professor of Mathematics, University of Rochester. Cloth, vii+151 pages, 1950. Dover Publications, 1780 Broadway, New York. $2.45.

Topology of Manifolds, by Raymond Louis Wilder, Professor of Mathematics, University of Michigan. Cloth, ix+404 pages, 1949. American Mathematical Society, 531 W. 116th Street, New York. $7.00.

Miscellaneous

Albert Einstein: Philosopher-Scientist, Paul Arthur Schilpp, Editor. Cloth, xvi+781 pages, 1949. Library of Living Philosophers, Inc., 101 Fayerweather Hall, East, Northwestern University, Evanston, Illinois. $8.50.

Modern Business Arithmetic, A Text-Workbook for Colleges, by Thomas M. Dodds, Dean of Instruction, Bryant and Stratton Business Institute; and Clyde Beighey, Head, Department of Business Education, Western Illinois State College. Paper, viii+280 pages, 1950. The Gregg Publishing Company, 270 Madison Avenue, New York. $2.20.

Process and Unreality, by Harry Kohlsaat Wells. Cloth, xiv+211 pages, 1950. King's Crown Press, Columbia University, New York. $3.00.

Science Is a Sacred Cow, by Anthony Standen, Polytechnic Institute of Brooklyn. Cloth, 219 pages, 1950. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York. $2.75

REVIEWS

Edited by J. Stipanowich, *Western State College, Macomb, Illinois*

Plane and Spherical Trigonometry. M. Richardson. 343+xxiii pp. The Macmillan Company 1950. With tables $3.75, without tables $3.40.

The distressed trigonometry teacher whose students cannot add fractions may be glad to see Dr. Richardson's "Plane and Spherical Trigonometry" which has a "Preliminary Review" compactly, but clearly, setting forth ideas from algebra and geometry which the student may need to review. This review may be studied through by the whole class if desirable, or it may be used for reference by the class or by individuals as it is needed.

Other points about the book that please a teacher are:

(1) A short though valuable section entitled "A little logic" summarizes principles of logic which are applied later—for instance, to making clear what operations are justified in proving trigonometric identities.

(2) The explanations are unusually clear and mathematically correct.

(3) The various cases of a discussion are systematically set down (Case 1, Case 2, etc.) and thoroughly covered.

(4) Historical background is woven into the discussion, not set apart to be skipped.

(5) Interest is aroused by frequent reference to what is coming later.

(6) The author seems to think of and clear up in a note every difficulty or special case about which the students need warning.

(7) The methods used in computing trigonometric tables are described in a way the student can understand.

(8) The work is arranged so the teacher has a choice of starting with the right triangle or with the general angle.

(9) An attempt is made to introduce not too many concepts at once.

(10) Enrichment material which may be omitted is clearly marked as such.

(11) Spherical trigonometry is more systematically and more completely treated than usual.—Henry Swain, New Trier Township High School, Winnetka, Illinois.

Geometry for Advanced Pupils. E. A. Maxwell. 176 pp. Oxford University Press, New York, 1950. $2.25.

The pattern of modern Euclidean geometry for college students has been set by such texts as Casey's and Altshiller-Court's. It is refreshing to have a new text with a novel presentation of the subject, and with the inclusion of some projective geometry. The text covers all the usual content of college geometry, has a wealth of riders (original exercises) and a fair introduction to concepts of projective geometry.

Each separate presentation begins with a configuration of which there are 47. Before the figuration is a development of important properties of the figure, without any formal enunciation of a theorem. There then follow a set of problems for solution, all relating to the given configuration. A student thus obtains a thorough, deep, and whole view of a large configuration rather than a catalogue of many varied relationships in many figures. The text promises to give a real insight into the nature of modern geometry as a closely integrated structure rather than a mere series of theorems.—Howard F. Fehr, Teachers College, Columbia University, New York City.

Mathematics for the General Reader. E. C. Titchmarsh. F. R. S. 160 pp. Hutchinson's University Library. Longmans, Green and Company Inc., New York, 1950. $1.60.

This book is intended to be a popularization of elementary mathematics for the layman. It assumes some earlier acquaintance with the usual secondary school algebra and geometry. The treatise, however, starts with counting and continues through arithmetic, algebra, coordinates, series, trigonometry, and the differential and integral calculus. To cover all of this in 160 pages and to achieve some clarity of exposition is no mean feat. For one who "knows", the exposition is remarkably clear, concise, pertinent and simple reading. For high school teachers of mathematics the book is excellent professional reading matter.

The book receives its compactness through the able selection of only the necessary essential concepts. However the rigor is frequently too great for non-technical readers. For example, the complete explanation of the Dedekind section is given in one short paragraph. The number system is extended by the use of ordered number pairs. There are no exercises for practice. The general reader will find this, as most other so-called popular mathematics texts, rather taxing on his mental facilities.—HOWARD F. FEHR.

The Skeleton Key of Mathematics. D. E. Littlewood. 138 pp., Hutchinson's University Library. Longmans, Green and Company Inc., New York, 1950. $1.60.

This book is an attempt to give a survey of modern abstract algebra that is somewhat deeper than popular science, but yet general enough to give the nonspecialist an insight into its methods. Accordingly, many details are omitted, no exercises are given, but the reader is urged from time to time to reconstruct the writing of the author. The method of abstraction is stressed throughout. The text opens with an introduction to the theory of numbers, then proceeds to algebraic fields, the theory of groups, algebraic geometry, matrices, invariants, algebras, and symmetric and continuous groups.

The writing is clear and concise. The introduction is unhurried and illustrated by simple numerical examples. However the more abstract material of the later chapters calls for considerable study and it is doubtful that an initiate can follow all of the development. The book can serve as an excellent review for one who has been away from his study of modern algebra. It can give an insight to modern methods for one acquainted with mathematics and logic, but it can hardly serve as a text for introducing this subject.—HOWARD F. FEHR.

Geometrical Constructions with a Ruler Given a Fixed Circle with Its Center. Jacob Steiner. Translated by Marion Stark and edited by Raymond Clare Archibald. 5+88 pp. Scripta Mathematica, New York, 1950. $2.00.

This is the first English translation of the first edition of Steiner's work which was published in German in 1833. The book opens with a short historical account of geometric construction and a sketch of Steiner's life and mathematical contributions. The text deals with the problem "Is it possible to make all Euclidean constructions by the use of a ruler alone, given a fixed circle and its center?" Steiner proves that it is possible and thus supplements Mascheroni's proof that all constructions are also possible by the use of compasses alone. The work is in three chapters, the first dealing with rectilinear figures, the second with the circle, and the third with the proof of the general problem.

Many of the theorems proved in the book are now a recognized part of modern geometry. These deal with harmonic ranges, poles and polars, centers of similitude, radical axes, etc. The third chapter is especially significant to students of geometry for the unique use of the previously established theorems in establishing the usual construction of plane geometry. All these constructions ultimately reduce to locating points which are the intersection of a line and a circle. The book is a welcome addition to references in the undergraduate study of modern college geometry.—HOWARD F. FEHR.

The Science of Chance. From Probability to Statistics. Horace C. Levinson. 9+348 pp. Rinehart and Company, New York, 1950. $2.00.

This book is a revision and expansion of the author's 1938 book "Your Chance to Win." It is written in simple and clear style and all principles are illustrated and explained by concrete examples from everyday life. The reader is shown the various fallacies concerning luck and chance events, and lead to general theorems on probability. At no time is the reader overtaxed with mathematical principles and operations. All that is necessary to understand the book is a fair amount of arithmetic and common reasoning ability.

The purpose of the book is to introduce the reader to those concepts of probability and statistics necessary to understand the modern use of these knowledges in science and everyday living. The author succeeds largely due to his unerring ability to stay within the readers' field of experience and to use only relevant exercises. The book is an excellent companion to every introductory course in statistics.—HOWARD F. FEHR.

Know Your National Council Representative

By Kenneth E. Brown

Chairman of State Representatives, Mathematics Department, University of Tennessee, Knoxville, Tennessee

R. A. Baumgartner

Illinois Representative

Mr. R. A. Baumgartner, Head of the Department of Mathematics at the Freeport High School, is serving his second year as the National Council Representative in Illinois; however, he has been an active member of the National Council for more than fifteen years. Mr. Baumgartner is a native of Illinois and began teaching in the Polo Community High School in 1935; he left Illinois in 1942 to serve four years in the Navy, and at the close of the Naval service he returned to the Freeport Community. Mr. Baumgartner reports that the first annual meeting of the Illinois Council of Teachers of Mathematics was held last October. Illinois led all states in membership in May, 1950. It is hoped that this state organization will increase in members and contributions to mathematics education.

Delaware Representative

Mrs. Florence E. Loose, in addition to performing the duties incumbent upon a mother of three children, has found time to participate in many professional activities. She is a past president of the Wilmington Teachers Association, and has served on many professional committees in addition to participating in the Mathematics Workshop at Duke University and in service training programs in Wilmington. The Mathematics Teacher has made a contribution to her success in teaching mathematics in the high school. In fact it is reported that she has one clothes closet partly filled with back numbers of The Mathematics Teacher to which she often refers for ideas that enrich her teaching. Her present address is 65 West Salisbury Drive, Edgemoor Terrace, Wilmington, Delaware.

Florence E. Loose

Program
The Eleventh Christmas Meeting
The National Council of Teachers of Mathematics

The University of Florida, Gainesville, Florida
December 27, 28 and 29, 1950

WEDNESDAY, DECEMBER 27, 1950

12:00 NOON–5:00 P.M. Registration

1:00 P.M.–5:00 P.M. Trip (arranged by local committee)

7:30 P.M. Reception or Open Meeting

THURSDAY, DECEMBER 28, 1950

8:00 A.M.–6:00 P.M. Registration

9:30 A.M.–10:30 A.M. General Session

Presiding: H. W. Charlesworth, East High School, Denver, Colorado

General Address: What Is Good Mathematics Teaching?
E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois

10:45 A.M.–12:00 NOON. Sectional Meeting (Elementary Section)

Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers
W. I. Layton, Stephen F. Austin State College, Nacogdoches, Texas

A Way of Determining When Different Meanings in Arithmetic Should Be Taught
J. T. Johnson, Chicago Teachers College, Chicago, Illinois

10:45 A.M.–12:00 NOON. Sectional Meeting (Secondary Section)

The Sandpile of Mathematics
W. P. Fulton, Crozier Technical High School, Dallas, Texas

The Teaching of Locus
Virgil S. Mallory, State Teachers College, Montclair, New Jersey

10:45 A.M.–12:00 NOON. Sectional Meeting (College Section)

How Can We Make the Teaching of Mathematics More Meaningful?
B. P. Reinsch, Florida Southern College, Lakeland, Florida

New Ways for Old
M. W. Keller, Purdue University, Lafayette, Indiana

10:45 A.M.–12:00 NOON. Sectional Meeting (Teacher Training Section)

Space and Geometry—Its Place in an Effective Teacher-Training Program.
Aaron Bakst, New York University, New York, N. Y.

How the Teaching of the Solutions of Exercises and Problems by Types Interferes With the Learning Processes
Nathan Lazar, Ohio State University, Columbus, Ohio

12:15 P.M. Get-Acquainted Luncheon, Program by Local Committee

2:00 P.M.–4:00 P.M. Discussion Groups

Group A1
Topic: Teaching Students to Solve Word Problems in High School Mathematics
Leader: W. P. Fulton, Crozier Technical High School, Dallas, Texas

Group A2
Topic: Curriculum Revision in Mathematics
Leader: D. W. Snader, University of Illinois, Urbana, Illinois

Group A3
Topic: A Course in Consumer Mathematics for Seniors in High School
Leader: Charlotte Carlton, Miami Beach High School, Miami Beach, Florida

Group A4
Topic: Undergraduate Training for Prospective Teachers of Mathematics
Leader: Henry A. Robinson, Agnes Scott College, Decatur, Georgia

Group A5
Topic: Guidance in Mathematics
Panel: Chairman, Veryl Schult, Wilson Teachers College, Washington, D. C.; Elinor Douglas, Woodrow Wilson High School, Washington, D. C.; Ruth Lane, Woodrow Wilson High School, Washington, D. C.; John R. Mayor, University of Wisconsin, Madison, Wisconsin; Agnes Motyka, Amidon-Greenleaf Elementary School, Washington. D. C.; Mary Potter, Public Schools, Racine, Wisconsin; Dorothy Wilson, Stuart

Junior High School, Washington, D. C.

Group A6
Topic: Contributions to Enliven Interest in Junior High School Mathematics
Leader: Mary C. Rogers, Roosevelt Junior High School, Westfield, N. J.

Group A7
Topic: Steps in Planning a Good Senior High School Functional Mathematics Program
Leader: Howard L. Gallant, Hillsborough High School, Tampa, Florida

Group A8
Topic: Geometric Models as an Enrichment Device at the Secondary and Junior College Level
Leader: J. M. Plant, Florida State University, Tallahassee, Florida

2:00 P.M.–5:00 P.M. Mathematics Films
Z. L. Loflin, Southwestern Louisiana Institute, Lafayette, Louisiana

2:00 P.M.–4:00 P.M. Sectional Meeting (Elementary Section)
Making the Study of Number Glow
Ida Mae Heard, Southwestern Louisiana Institute, Lafayette, Louisiana
Arithmetic, Algebra and Geometry, Each as an Aid in the Study of the Others
W. W. Rankin, Duke University, Durham, North Carolina

2:00 P.M.–4:00 P.M. Sectional Meeting (Secondary Section)
Once Upon a Time (The Use of Historical Material in the Teaching of Secondary Mathematics)
Catherine Dorsey, University High School, University of West Virginia, Morgantown, West Virginia
Teaching the New Geometry
C. N. Shuster, State Teachers College, Trenton, New Jersey

2:00 P.M.–4:00 P.M. Sectional Meeting (Teacher Training Section)
A Proposed Program for the Training of Secondary Mathematics Teachers
Orval A. Phillips, Mississippi Southern College, Hattiesburg, Mississippi
The Selection of Teachers of Mathematics Through a Competitive Examination System
Joseph Jablonower, The Board of Examiners, Board of Education, New York, N. Y.

2:00 P.M.–4:00 P.M. Sectional Meeting (College Section)
A Program for Teaching Fellows
Phillip S. Jones, University of Michigan, Ann Arbor, Michigan
The Ideal Teacher of College Mathematics
Wllliam A. Gager, University of Florida, Gainesville, Florida
The New Instructor and the Department of Mathematics
Wilson L. Miser, Vanderbilt University, Nashville, Tennessee

6:30 P.M. Banquet
Presiding: F. W. Kokomoor, University of Florida, Gainesville, Florida
Address of Welcome:
Response: H. W. Charlesworth, President, N.C.T.M.
Address: "Defeatism in Education," C. C. MacDuffee, University of Wisconsin, Madison, Wisconsin

FRIDAY, DECEMBER 29, 1950

8:30 A.M.–3:00 P.M. Registration

8:30 A.M.–9:30 A.M. Affiliated Groups
J. R. Mayor, Chairman, University of Wisconsin, Madison, Wisconsin

8:30 A.M.–9:30 A.M. Mathematics Films
Z. L. Loflin, Southwestern Louisiana Institute, Lafayette, Louisiana

9:30 A.M.–10:30 A.M. General Session
Presiding: James H. Zant, Oklahoma A. and M. College, Stillwater, Oklahoma
Address: The Psychology of Learning Applied to Classroom Teaching of Mathematics.
Howard F. Fehr, Columbia University, New York, N. Y.

10:45 A.M.–12:00 NOON. Sectional Meeting (College section)
Measurement and Dimensionality from a Postulational Point of View
Aaron Bakst, New York University, New York, N. Y.

10:45 A.M.–12:00 NOON. Sectional Meeting (History and Mathematics)
Mathematical Footnotes to History
Walter H. Carnahan, Purdue University, Lafayette, Indiana

10:45 A.M.–12:00 NOON. Discussion Groups

Group B1
Topic: Slides and Devices as Aids to Learning (How to Make Them and How to Use Them)
Leader: Agnes Herbert, Clifton Park Junior High School, Baltimore, Md.

Group B2
Topic: A Graduate Program for the Training of Mathematics Teachers

Leader: Orval Phillips, Mississippi Southern College, Hattiesburg, Miss.

Group B3
Topic: The Florida Program on the Functional Approach to the Teaching of Mathematics
Leader: Lilla C. Lyle, Senior High School, Miami Beach, Florida

Group B4
Topic: Teaching Students to Read in Mathematics
Leader: Virginia Felder, Mississippi Southern College, Hattiesburg, Miss.

Group B5
Topic: How Should We Teach Computation with Approximate Numbers and Avoid Pretended Accuracies with Computed Results?
Leader: B. P. Reinsch, Florida Southern College, Lakeland, Florida

Group B6
Topic: General Mathematics in the College
Leader: William B. Higgins, Teachers College, Columbia University, New York, N. Y.

Group B7
Topic: Making Mathematics Interesting to Junior High School Students
Leader: Edith Woolsey, Sanford Junior High School, Minneapolis, Minn.

Group B8
Topic: What Shall Be the Mathematical Offering and Grade Placement in the Small High School? (200 or Less)
Leader: Gladys M. Thomason, President, Georgia Mathematics Teachers, Sandersville, Ga.

Group B9
Topic: Track 1 Mathematics—What Shall It Be for College-Going Students?
Panel: Chairman, Jackson B. Adkins, Phillips Exeter Academy, Exeter, New Hampshire; A. A. Bennett, Brown University, Providence, Rhode Island; William A. Gager, University of Florida, Gainesville, Florida; Carl N. Shuster, State Teachers College, Trenton, New Jersey

12:15 P.M. Luncheon

2:00 P.M.–4:00 P.M. Sectional Meeting (Elementary Section)
Organization—the Key to Learning in Elementary Mathematics
John R. Mayor, University of Wisconsin, Madison, Wisconsin
Understanding—the Key to Generalization in Elementary Mathematics
Agnes Motyka, Principal, Amidon-Greenleaf Elementary School, Washington, D. C.

2:00 P.M.–4:00 P.M. Sectional Meeting (Junior High Section)
Mathematics Plus
Doris Johnson, Clearwater Junior High School, Clearwater, Florida
What Have Your Pupils Learned—Suggestions for Evaluation
F. G. Lankford, Jr., University of Virginia, Charlottesville, Virginia

2:00 P.M.–4:00 P.M. Sectional Meeting (Senior High Section)
Meaningful Versus Mechanical Algebra
H. C. Christofferson, Miami University, Oxford, Ohio
Using Mathematics Now vs. Using Mathematics in the Future
J. J. Spencer, St. Augustine High School, St. Augustine, Florida

2:00 P.M.–4:00 P.M. Sectional Meeting (College Section)
A Film—Mathematics as a Language
H. C. Trimble, Florida State College, Tallahassee, Florida
General Mathematics on the Junior College Level
Kenneth E. Brown, University of Tennessee, Knoxville, Tennessee
Minimum Non-Technical Mathematics for College Students
F. W. Kokomoor, University of Florida, Gainesville, Florida

2:00 P.M.–4:00 P.M. Sectional Meeting (Teacher Training Section)
R-H (Real Help) Factors in Teacher Training
Frank C. Bolser, Florida State University, Tallahassee, Florida
Emphasizing Teacher Training in Undergraduate Mathematics
H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa

Program Committee

Chairman, James H. Zant; Aaron Bakst; Walter H. Carnahan; H. C. Christofferson; J. R. Clark; H. F. Fehr; Virginia Felder; W. P. Fulton; W. A. Gager; Agnes Herbert; E. H. C. Hildebrandt; Julia Mae Hoag; P. S. Jones; H. T. Karnes; John R. Mayor; Bess Patton; Mary C. Rogers; Veryl Schult; D. W. Snader; H. W. Syer; H. C. Trimble; May L. Wilt; Edith Woolsey.

Announcements

Registration

There will be a registration fee of 50¢ each for members of the National Council, members of the Mathematical Association of America, and for teachers in the elementary schools. The fee for non-members and visitors is $1.50 each. Undergraduate students sponsored by faculty members, relatives of members, invited speakers who are not members, members of the press, and exhibitors are admitted without payment of registration fee, but should register at the registration desk in order to obtain identification badge. Registration will be in the P. K. Yonge Laboratory School, Room 134.

Housing Accommodations

Housing accommodations will be provided in the University dormitories from Tuesday through Saturday nights, December 26–30, at $2.00 per person per night in double rooms and $2.50 per person per night in single rooms. This fee includes linen and maid service. All who expect to reside in dormitories must engage and pay for rooms before December 10, 1950.

Several hotels are available in the city for those who prefer not to live in dormitories. Among these are: Hotel Thomas, White House Hotel, Arlington Hotel, Plaza Hotel, Buena Vista Hotel, Casa Loma Lodge, Commercial Hotel, Central Hotel, Primrose Grill, and Gilbert Hotel. There are also, within or near the outskirts of the city, several motels which can furnish some rooms. Reservations in any of these should be made directly with the hotel concerned and well in advance of the meeting.

Those desiring dormitory housing on nights other than those indicated above should write to the Director of Housing, University of Florida. It may be possible to accommodate them.

Dining Accommodations

Meals will be available to all in the University Cafeteria from 12 Noon, Tuesday, December 26, through break-

REGISTRATION AND RESERVATION FORM

Please fill out and mail with remittance to Mr. H. Kenneth Lewis, Treasurer, Department of Mathematics, Peabody 104, University of Florida, Gainesville, Florida, so as to reach him not later than Dec. 10, 1950.

Name (Please Print)

Mr., Miss, Mrs. _____
Last First Name Initial

Home Address _____
Street and Number City Zone State

School with which connected, if any _____
Name

_____
City State

Member of N.C.T.M. Yes ( ) No ( )

Member of M.A.A. Yes ( ) No ( )

Student ( )

Please check your field(s) of interest:

( ) Elem. ( ) Jr. H.S. ( ) High School ( ) Jr. Col.

( ) College ( ) Teacher Training ( ) Supervision

( ) Other (please state)

Registrations

Number of 50¢ fees (members and elementary teachers) sent _____

For whom? (give names and addresses) _____,

_____, _____

Number of $1.50 fees (non-members) sent _____

For whom? (give names) _____,

_____, _____

Total amount enclosed for registration fees $_____

fast, Sunday, December 31, 1950.

On Wednesday, December 27, at 7:30 P.M., a reception for members of the National Council, the Florida Council, other affiliated organizations, and friends will be held in Mallory Hall at S.W. 5th Avenue and 13th Street.

At 12:15 P.M., Thursday, December 28, a "Get Acquainted" luncheon will be served in the east wing of the University Cafeteria. The program for this luncheon will be under the direction of the Florida Council of Teachers of Mathematics.

The National Council Banquet will be held at 6:30 P.M. on Thursday, December 28, in the east wing of the University Cafeteria. The price of a banquet ticket is $2.50, and must be ordered and paid for before December 10, 1950.

Sightseeing Trips

At 1:00 P.M., on Wednesday, December 27, chartered Greyhound busses, as needed, will leave for Silver Springs from the front of P. K. Yonge Laboratory School (corner of S.W. 5th Avenue and 13th Street). The maximum capacity of each bus is 36 passengers, but a bus will make the trip if at least 25 can be secured. At the Springs, opportunity is available for swimming, glass-bottom boat rides, a jungle cruise, and a visit to the Ross Allen Reptile Institute. The return to Gainesville will be made through Cross Creek, home of Marjorie Kinnan Rawlings, author of *South Moon Under*, *The Yearling*, and other well-known books. Round trip transportation will be $2.00 per person, for which reservations must be made and paid for before December 10, 1950. Costs other than transportation are extra. The round trip distance is about 100 miles, and the time of arrival back in Gainesville will be about 5:00 P.M.

On Thursday, December 28, 1950, chartered busses, as needed, will take passengers to Daytona Beach, Marineland, St. Augustine, and the Penney Farms Memorial Community. The approximate program for the day is as follows: Leave Gainesville 7:00 A.M., arriving Daytona Beach 9:00 A.M., having a one-hour visit at the beach. Leave Daytona Beach 10:00 A.M., arriving at Marineland about

RESERVATIONS

Dormitory Accommodations

Single rooms at $2.50 each per night. Give name, sex, and check nights room is needed:

Name	M.	F.	Tues.	Wed.	Thurs.	Fri.	Sat.
1)							
2)							

Double rooms at $2.00 each per night. Please check requested information:

Names	Relationship			Nights needed				
	Couple	M.	F.	Tues.	Wed.	Thu.	Fri.	Sat.
and								
and								

Total amount enclosed for dormitory rooms $_____

National Council Banquet. Number of tickets desired ($2.50 each) _____

Amount encl. $_____

For whom? (give names) _____, _____

Silver Springs Trip. Number of tickets desired ($2.00 each) _____

Amount encl. $_____

For whom? (give names) _____, _____

Daytona Beach-St. Augustine Trip. Tickets desired ($5.00 each) _____

Amount encl. $_____

For whom? (give names) _____, _____

10:30 A.M. (Marineland Studios offer the visitor a fine opportunity to see many forms of marine life at close range. 11:00 A.M. is feeding period, and, therefore, one of the most interesting times to visit the Studios.) Leave Marineland Studios 11:30 A.M., arriving at St. Augustine about noon. Time will be allowed here for lunch and for visiting Castillo de San Marcos, the Alligator Farm, the old Slave Market, and other points of interest. Leave St. Augustine about 3:00 P.M. for Gainesville, crossing the St. Johns River at Green Cove Springs and stopping over at the Penney Farms Memorial Community enroute home. Arrive in Gainesville about 5:30 P.M. The round-trip transportation cost of this trip will be $5.00 per person, for which reservations must be made and paid for before December 10, 1950. Costs other than transportation are extra. The total distance of this trip is about 235 miles.

Exhibits

There will be an exhibition of various types of classroom materials, such as mathematical models, instructional charts and graphs, and other teaching aids.

Invitation is hereby extended to all to present materials suitable for exhibition. Anyone willing to present or suggest suitable materials for this collection should communicate in advance with Dr. Kenneth P. Kidd, P. K. Yonge Laboratory School, University of Florida, Gainesville, Florida.

Commercial exhibits of text books and teaching aids will be welcome. Inquiries for exhibit space of such materials should also be addressed to Dr. Kenneth P. Kidd.

Speakers' Supplies and Equipment

Speakers and other participants in the program who expect to have need for special blackboard space, projection equipment, or other materials not usually found in a classroom, should communicate with Mr. Allyn C. Litherland, Department of Mathematics, University of Florida, not later than December 15, 1950.

Films and Film Strips

There will be showings of films throughout the three days of meeting, according to the program schedule. Film producers who wish to show their products, and persons desiring to show special films while in attendance at the meeting, should communicate with Mrs. Annette Williams, Audio Visual Service, University of Florida Library, not later than December 1, 1950.

Transportation

Gainesville can be reached by plane over the Eastern Airlines, and by rail over the Atlantic Coast Line and the Seaboard Airline Railroad. Those who come by Seaboard may purchase their tickets to Waldo, Florida, which is 12 miles from Gainesville. University busses will meet them at Waldo, provided that notice of time of arrival is received by the Department of Mathematics before December 10, 1950. This bus service will also be available for the return trip. Gainesville can also be reached by bus over the Greyhound and Trailways lines. On account of the heavy tourist travel during this time of year, it is highly desirable that all train, bus, and plane travel reservations be made well in advance. Those driving their own cars will find excellent highways leading into Gainesville. U. S. roads 1, 17, 301, 129, 41, 27, and 90 all lead into the State from the north and west. Any Florida road map will show how to reach Gainesville from these highways.

Applications for Registration and Reservations

All applications for registration and reservations for housing, banquet, and sightseeing trips must be made on the form printed below or a similar one obtained by writing the Department of Mathematics, Peabody 104, University of Florida, Gainesville, Florida. Applications must be returned, with remittance by check or money order, so as to reach destination not later than December 10, 1950. Checks and money orders should be made out to Mr. H. Kenneth Lewis, Treasurer.

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Join the National Council of Teachers of Mathematics

Any person who is interested in the field of mathematics is invited to apply for membership in the Council which has for its object the advancement of mathematics teaching at all levels of instruction. Its activities include:

1. The publication of *The Mathematics Teacher* which appears monthly except in *June, July, August, and September.* The annual membership dues of $3.00 for individuals include a one year's subscription to *The Mathematics Teacher* beginning with the issue you may designate. The subscription price to others (libraries, schools, colleges, etc.) is $5.00 per year.

2. The publication of yearbooks. The only available yearbooks at present are

The Third: *Selected Topics in Teaching Mathematics*
The Fourth: *Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910*
The Sixth: *Mathematics in Modern Life*
The Eighth: *The Teaching of Mathematics in Secondary Schools*
The Fourteenth: *The Training of Mathematics Teachers of Secondary Schools*
The Fifteenth: *The Place of Mathematics in Secondary Education*
The Sixteenth: *Arithmetic in General Education*
The Eighteenth: *Multi-Sensory Aids in Teaching Mathematics*
The Nineteenth: *Surveying Instruments—Their History and Classroom Use*
The Twentieth: *The Metric System of Weights and Measures*

These may be obtained for $3 each, postpaid. Send orders and checks directly to Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York 27, New York.

3. The conducting of research through committees on problems at all levels of instruction. One recent report is entitled "Guidance Pamphlet in Mathematics." Copies may be obtained for 25¢ each or 10¢ each if ordered in lots of ten or more. Send orders and remittance to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.

4. Cooperation with over forty affiliated groups which average three or four meetings annually. If there is one in your area, be sure to join it.

5. Holding at least four conventions annually. Watch *The Mathematics Teacher* for dates and locations.

If you are not a member, fill out the application blank below (or a copy of it) today and join with thousands of other mathematics teachers who are members of the only national organization devoted solely to the interests of mathematics teachers in Elementary and Secondary Schools, Junior Colleges and Teacher Education.

If you are a member, will you tell others, who are not members, of the work of the National Council and make the application blank below (or a copy of it) available to them for their use?

MEMBERSHIP BLANK

I hereby apply for membership in The National Council of Teachers of Mathematics. I enclose $3.00 for my dues and would like to have my subscription to *The Mathematics Teacher* begin with the ..... issue. (If this is a renewal, please indicate.)
month, year

Name (Please print)

Mr., Mrs., Miss ..... (check one) First Name Initial Last Name

School Address ..... Institution City State or Province

Home Address ..... Street No. City Zone No. State or Province

Place an X in front of the address to which *The Mathematics Teacher* is to be sent.

Please check the school classification of your present teaching position:

( ) Elem.; ( ) Jr. H.S.; ( ) H.S.; ( ) Jr. Col.; ( ) Col.; ( ) Tchr. Educ.; ( ) Supervision; Other ..... and place checks above the grade(s) and subject(s) you are teaching this year:

K, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, others ..... Arith, Gen Math I, Gen Math II, Alg I, Alg II, Pl Geom, Sol Geom, Trig, Col Alg, Others ..... This application and your remittance should be sent to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.